The Price of Macroeconomic Uncertainty[1]

with Tenuous Beliefs

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[1] Based on joint work Thomas J. Sargent

Why look a growth-rate uncertainty?

Motivation

- Macroeconomists speculate about the potential permanence of secular stagnation
- Economic historians debate the future of technological progress

Tools of analysis

- Statistical decision theory
- Asset pricing
- Valuation counterparts to impulse response functions

Recursive valuation

- Use a recursive utility model (see Koopmans, Kreps & Porteus, Epstein & Zin, …) to highlight how uncertainty about future events affects current asset valuation.
- Explore ways in which expectations and uncertainty about future growth rates influence risky claims to consumption.
- The *forward-looking* nature of the recursive utility model provides an additional channel through which *perceptions* about the future matter. (Bansal-Yaron and many others.)

Recursive utility

Consider the aggregator specified in terms of C_t the current period consumption and U_t the continuation utility :

$$U_t = \left([1 - \exp(-\delta\epsilon)](C_t)^{1-\rho} + \exp(-\delta\epsilon) \left[\mathcal{R}_t(U_{t+\epsilon}) \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

where

$$\mathcal{R}_t \left(U_{t+\epsilon} \right) = \left(\mathcal{E} \left[(U_{t+\epsilon})^{1-\gamma} | \mathcal{F}_t \right] \right)^{\frac{1}{1-\gamma}}$$

adjusts the continuation utility $U_{t+\epsilon}$ for risk.

- ▷ $\frac{1}{\rho}$ is the elasticity of intertemporal substitution
- ▷ δ is a subjective discount rate.
- ▷ ϵ is the decision interval.

Models of asset valuation

Two channels:

- ▷ Stochastic growth modeled as a process $G = \{G_t\}$ where G_t captures growth between dates zero and t.
- ▷ Stochastic discounting modeled as a process $S = \{S_t\}$ where S_t assigns risk-adjusted prices to cash flows at date t.

Date zero prices of a payoff G_t are

$$\pi = \mathcal{E}\left(S_t G_t | \mathcal{F}_0\right)$$

where $\mathcal{F}_0$ captures current period information.

Stochastic discounting reflects investor preferences through the intertemporal marginal rate of substitution for marginal investors.

Stochastic discount factor

$$\frac{S_{t+\epsilon}}{S_t} = \exp(-\delta\epsilon)\left(\frac{C_{t+\epsilon}}{C_t}\right)^{-\rho}\left[\frac{U_{t+\epsilon}}{\mathcal{R}_t(U_{t+\epsilon})}\right]^{\rho-\gamma}$$

- Continuation utilities give a structured way to enhance the impact of the perceptions about the future.
- Special case: power utility sets $\rho = \gamma$: no forward-looking component.
- Special case: unitary elasticity of substitution sets $\rho = 1$:

$$\left[\frac{U_{t+\epsilon}}{\mathcal{R}_t(U_{t+\epsilon})}\right]^{1-\gamma} = \frac{(U_{t+\epsilon})^{1-\gamma}}{E\left[(U_{t+\epsilon})^{1-\gamma}|\mathcal{F}_t\right]}$$

  has conditional expectation equal to unity. Equivalent interpretation as distorted beliefs.
- Multiply to compound over multiple periods.

Risk-return tradeoffs

Dynamic asset pricing through altering cash flow exposure to shocks.

- ▷ Study implication on the price today of changing the exposure tomorrow on a cash flow at some future date.
- ▷ Represent shock price elasticities by normalizing the exposure and studying the impact on the logarithms of the expected returns.
- ▷ Construct pricing counterpart to impulse response functions.

Unpack the term structure of risk premia!

Counterparts to impulse response functions pertinent to valuation:

- ▷ shock-exposure elasticities
- ▷ shock-price elasticities

These are the ingredients to risk premia, and they have a term structure induced by the changes in the investment horizons.

Hansen-Scheinkman (*Finance and Stochastics*), Borovička and Hansen ((*Journal of Econometrics*), Borovička-Hansen-Scheinkman (*Mathematical and Financial Economics*)

Construct elasticities

- Construct shock elasticities as counterparts to impulse response functions

- Use (exponential) martingale perturbation $D_{(\tau,\tau+s)}$ to an underlying positive (multiplicative) process M where:

$$d \ln M_t = \mu_m(X_t)dt + \sigma_m(X_t) \cdot dZ_t$$

$$\epsilon_m(x,t,\tau) := \lim_{s \downarrow 0} \frac{d}{ds} \log \mathcal{E}\left[\frac{M_t}{M_0} D_{(\tau,\tau+s)} | X_0 = x\right]$$

- Apply to a cash-flow G_t and stochastic discount factor S_t
  - shock exposure elasticity $\epsilon_g(x,t,\tau)$;
  - shock cost elasticity $\epsilon_{sg}(x,t,\tau)$;
  - shock price elasticity $\epsilon_g(x,t,\tau) - \epsilon_{sc}(x,t,\tau)$

In what follows $\tau = 0$ or $\tau = t$.

Interpret elasticities

Recall

$$\epsilon_m(x,t,\tau) = \lim_{s\downarrow 0} \frac{d}{ds} \log \mathcal{E}\left[\frac{M_t}{M_0} D_{(\tau,\tau+s)} | X_0 = x\right]$$

where $D_{(\tau,\tau+s)}$ is an exponential martingale perturbation.

Two interpretations:

- ▷ Change in probability measure - local impulse response
- ▷ Change in cash flow exposure - local risk risk return

Depend on current state, horizon, and date when the perturbation occurs.

A long-run risk example

Bansal Yaron

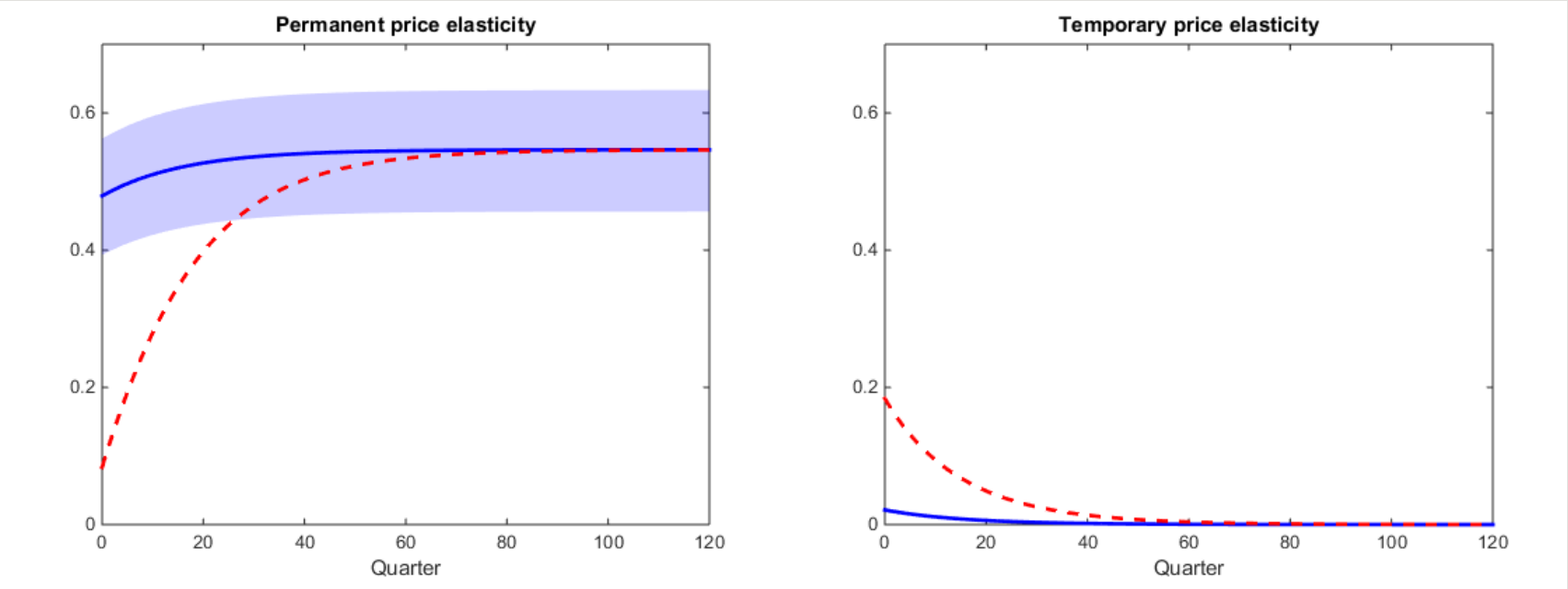
$$dZ_t = -.021 Z_t dt + \sqrt{V_t} \begin{bmatrix} .031 & -.015 & 0 \end{bmatrix} dW_t$$
$$dV_t = -.013 \left(V_t - 1\right) dt + \sqrt{V_t} \begin{bmatrix} 0 & 0 & -.038 \end{bmatrix} dW_t$$
$$dY_t = (.01)(.15 + Z_t) dt + (.01) \sqrt{V_t} \begin{bmatrix} .34 & .7 & 0 \end{bmatrix} dW_t$$

- Y_t is the logarithm of consumption
- Z_t captures predictability in macroeconomic growth rates
- V_t captures stochastic volatility
- 3 components of dW_t:
  - permanent shock
  - transitory shock
  - stochastic volatility shock

Interpretating the shocks

- Introduce production by extending an adjustment cost of model of Eberly and Wang.
- 3 shocks:
  - technology shock
  - investment productivity shock
  - common stochastic volatility shock
- Shocks have permanent consequences.

Shock-Price Elasticities



Recursive utility and Power utility. Bands depict .1 and .9 deciles.

Success?

- The mechanism relies on endowing investors with knowledge of statistically subtle components of the macro time series. Where does this confidence come from?
- Imposes stochastic volatility exogenously.
- Imposes large risk aversion.

Uncertainty components

- Risk -
  uncertainty within a model: unknown outcomes with known probabilities
- Ambiguity -
  uncertainty across models: unknown weights for alternative possible models
- Misspecification -
  uncertainty about models: unknown flaws of approximating models

Making Robustness Operational

- Explore a family of "posteriors/priors" used to weight structured models. Dynamic learning could play a central role. (Robust Bayesian analysis)
- Explore a family of alternative potential unstructured models or a class of perturbations to a benchmark model subject to constraints or penalization. Future perturbations may not be tied to the past making learning about them impossible. (Control theory and statistical origins.)

Use the decision problem to target the member of the family that has the largest utility consequences.

Worst-case Model of a Belief Distortion

- Apply the theory of two-person games. The decision maker optimizes taking as given the worst-case probability.
- Decentralize with a constrained worst-case probability.

Concerns about model misspecification look like belief distortions.

Good Thinking

> *In what circumstances is a minimax solution reasonable? I suggest that it is reasonable if and only if the least favorable initial distribution is reasonable according to your body of beliefs.*

from I. J. Good (1952) on rational decisions

- ▷ admissibility
- ▷ complete class theorem
- ▷ connect the *ex post* choice of priors with the decision problem

Underappreciated tension between this perspective and dynamic consistency

Two types of uncertainty

- ▷ families of structured models - confront unknown parameters and parameters that change over time
- ▷ families of unstructured models - confront potential model misspecification

Alternative probabilities

- Use positive martingales M^H to represent alternative probabilities relative to some baseline

$$dM_t^H = M_t^H H_t \cdot dW_t$$

  or

$$d \log M_t^H = H_t \cdot dW_t - \frac{1}{2} H_t \cdot H_t$$

  Change of measure induces a drift distortion $H_t dt$ in the underlying Brownian motion.
- M^S is a family of structured probability models where $H = S$.
- M^U denotes alternative unstructured probability models where $H = U$.

Robustness concerns illustrated

- Initial model

$$dY_t = (.01)\left(\widehat{\alpha}_y + \widehat{\beta} Z_t\right) dt + (.01)\sigma_x \cdot dW_t$$
$$dZ_t = \widehat{\alpha}_z dt - \widehat{\kappa} Z_t dt + \sigma_z \cdot dW_t$$

- W a Brownian motion
- Y aggregate macro indicator which we take to be log consumption
- Z generates "long-run risk" or growth rate uncertainty

Family of Restricted Models

- Parameters: $\alpha_y, \beta, \alpha_z, \kappa$
- Evolution:

$$dY_t = .01\left(\alpha_y + \beta Z_t\right)dt + .01\sigma_y \cdot dW_t^R$$
$$dZ_t = \alpha_z dt - \kappa Z_t dt + \sigma_z \cdot dW_t^R$$

- Construct drift distortion for the Brownian motion $dW_t = R_t dt + dW_t^R$ where $R_t = \eta(Z_t) \equiv \eta_0 + \eta_1 Z_t$ and where

$$\sigma = \begin{bmatrix} (\sigma_y)' \\ (\sigma_z)' \end{bmatrix},$$

and

$$\sigma\eta_0 = \begin{bmatrix} \alpha_y - \hat{\alpha}_y \\ \alpha_z - \hat{\alpha}_z \end{bmatrix} \quad \sigma\eta_1 = \begin{bmatrix} \beta - \hat{\beta} \\ \hat{\kappa} - \kappa \end{bmatrix}$$

Relative entropy

- entropy of unstructured relative to the baseline $M = 1$ probability:

$$\Delta\left(M^U; 1|\mathcal{F}_0\right) = \int_0^\infty \exp(-\delta t) E\left(M_t^U \log M_t^U \mid \mathcal{F}_0\right) dt$$
$$= \frac{\delta}{2}\int_0^\infty \exp(-\delta t) E\left(M_t^U |U_t|^2 \middle| \mathcal{F}_0\right) dt.$$

- entropy of unstructured relative to structured probability

$$\Delta\left(M^U; M^S \mid \mathcal{F}_0\right) = \frac{\delta}{2}\int_0^\infty \exp(-\delta t) E\left(M_t^U \mid U_t - S_t \mid^2 \middle| \mathcal{F}_0\right) dt.$$

Restraining structured models

Impose separability over time (Chen Epstein). Two motivations:

- ▷ dynamic consistency as in Epstein and Schneider
- ▷ time-varying parameters - Good

Omits statistical neighborhoods from consideration - not well suited to study model misspecification

Restraining structured models

- Compute relative entropy as a discounted integral using recursive methods

$$\frac{\delta}{2}\int_0^\infty \exp(-\delta t) E\left(M_t^S |S_t|^2 \middle| \mathcal{F}_0\right) dt.$$

  for $S_t = \eta(X_t)$. The outcome is the relative entropy value function ρ.

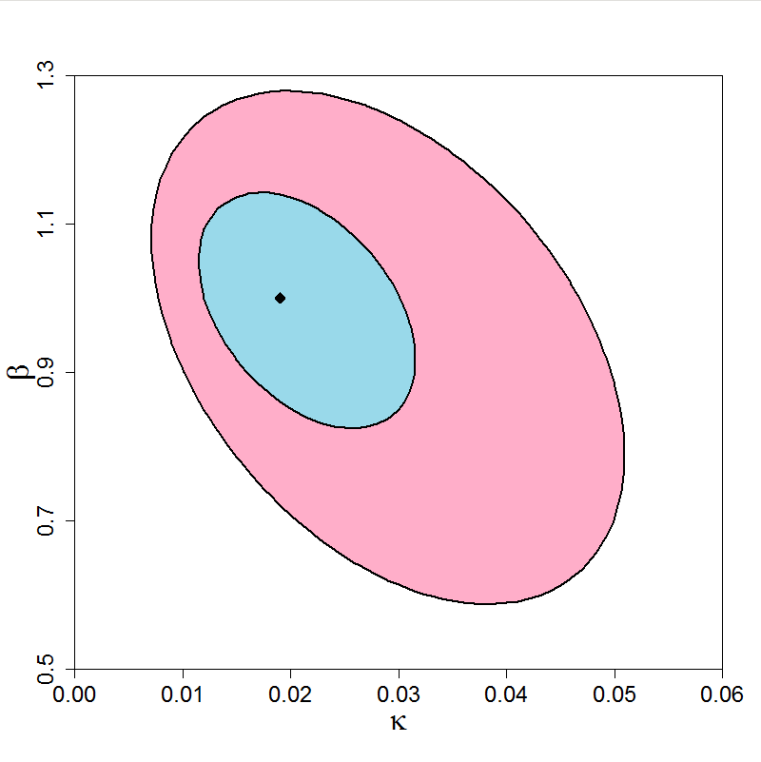
- Use $\delta = 0$ limiting value function to restrain structured models

$$\frac{\partial \rho}{\partial x}(x)\cdot[\widehat{\mu}(x) + \sigma(x)s] + \frac{1}{2}\text{trace}\left[\sigma(x)' \frac{\partial^2 \rho}{\partial x \partial x'}(x)\sigma(x)\right] + \frac{|s|^2}{2} \leq \frac{\mathsf{q}^2}{2}.$$

  ρ and q are specified *a priori*. $\frac{\mathsf{q}^2}{2}$ relative entropy and q is a measure of the magnitude of the drift distortion.

Slope uncertainty

$$dY_t = .01\left(\alpha_y + \beta Z_t\right)dt + .01\sigma_y \cdot dW_t^R$$
$$dZ_t = \alpha_z dt - \kappa Z_t dt + \sigma_z \cdot dW_t^R$$



Boundaries of two sets of alternative parameter values (β, κ).

Misspecified dynamics

- Introduce unstructured drift distortions U_t:

$$dW_t = U_t dt + dW_t^U$$

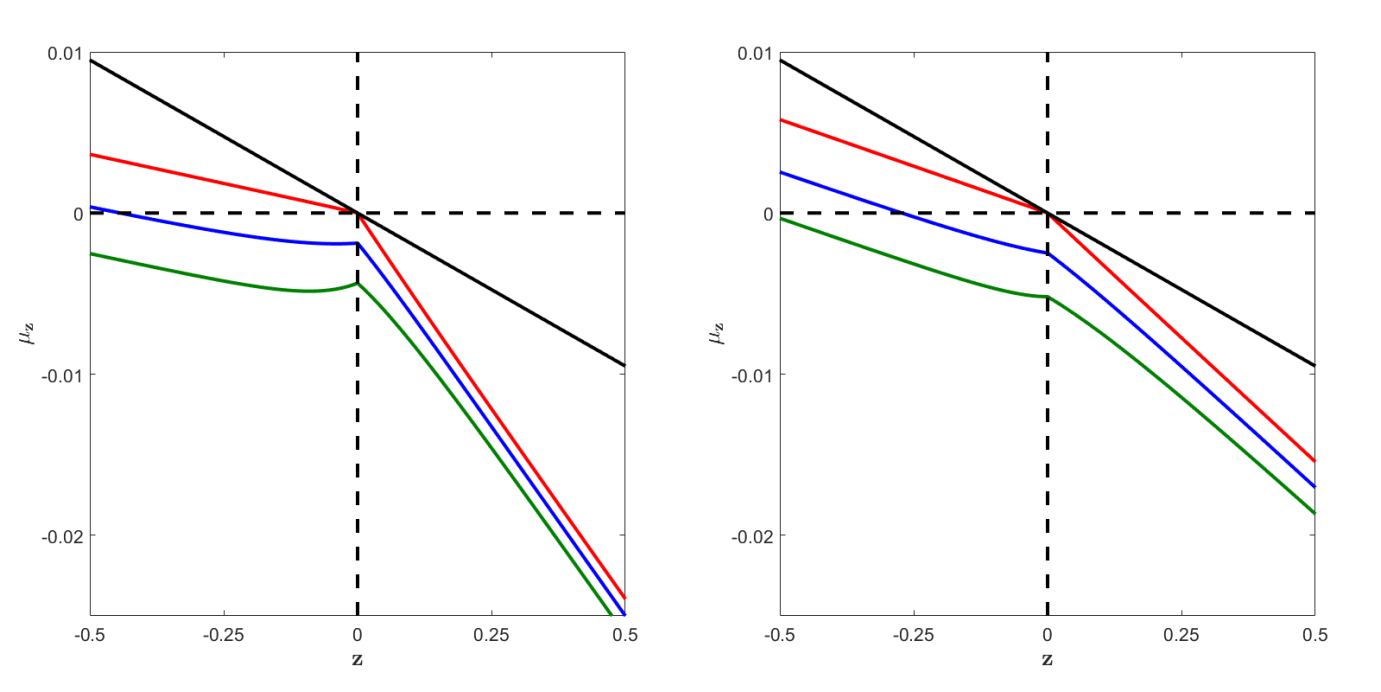
- Penalize using

$$\frac{\delta}{2}\int_0^\infty \exp(-\delta t) E\left(M_t^U |U_t - S_t|^2 \middle| \mathcal{F}_0\right) dt.$$

- Impose a quadratic penalty $|U_t - S_t|^2$ where S_t is one of the possible modeled drift distortions
- Minimize over the restricted family of S_t's

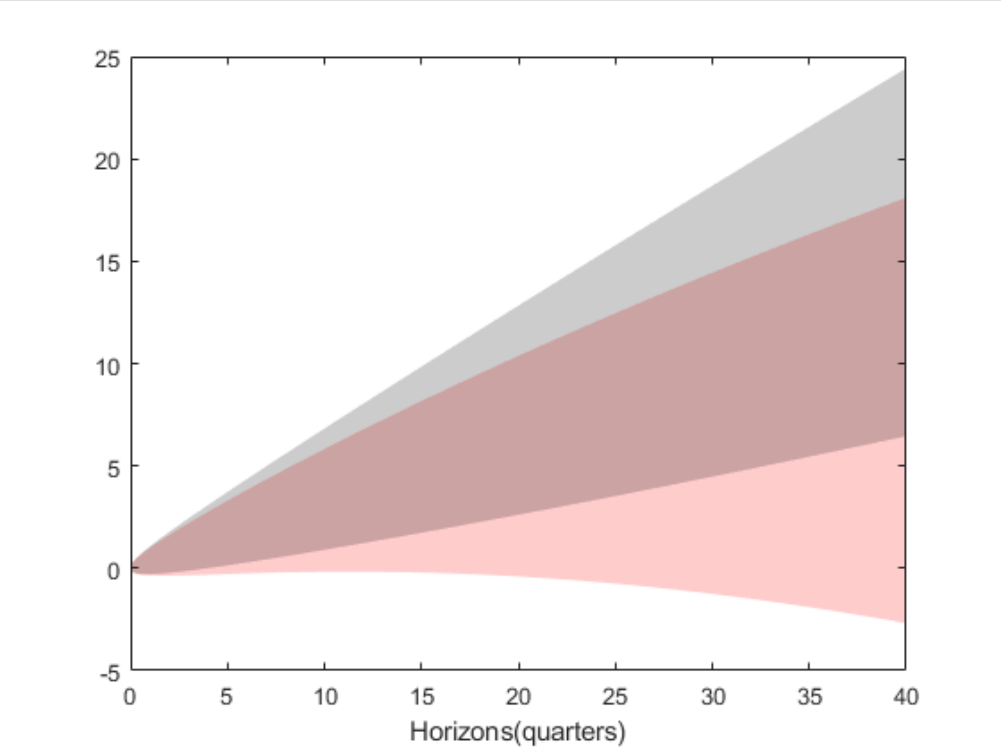
Special case of variational preferences of Marinacci (JET) *et al* and extends Hansen-Sargent (AER).

Local growth rate uncertainty



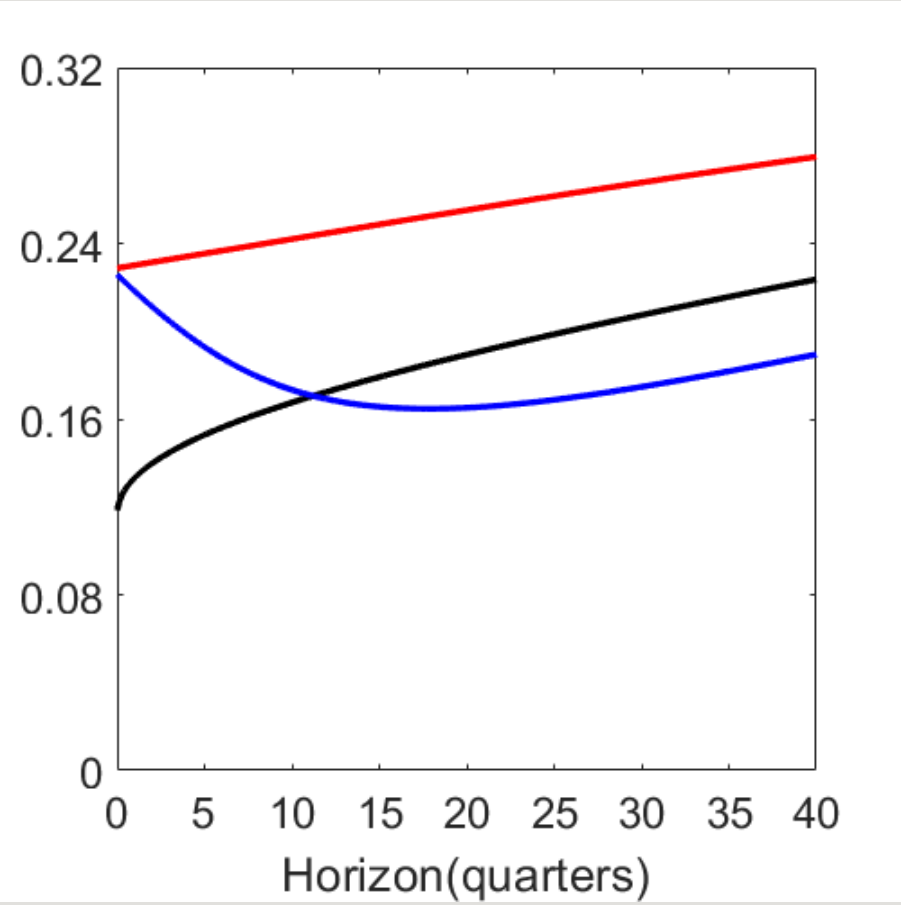
Growth rate drift functions. Left panels: larger baseline entropy. Right panels: smaller baseline entropy. **Black**: baseline model; **red**: worst-case structured model ; **blue**: transformed entropy = .1 ; and **green**: transformed entropy = .2.

Tilting Probabilities



Distribution of $Y_t - Y_0$ under the baseline model and worst-case model. The gray shaded region include includes .1- .9 deciles for the baseline model. The red shaded region includes the .1-.9 deciles for the worst case.

Term Structure of Uncertainty Prices



Shock price elasticities. Exposure change occurs at the same future date as the consumption payoff. **Black**: median of the Z stationary distribution **red**: .1 decile; and **blue**: .9 decile.

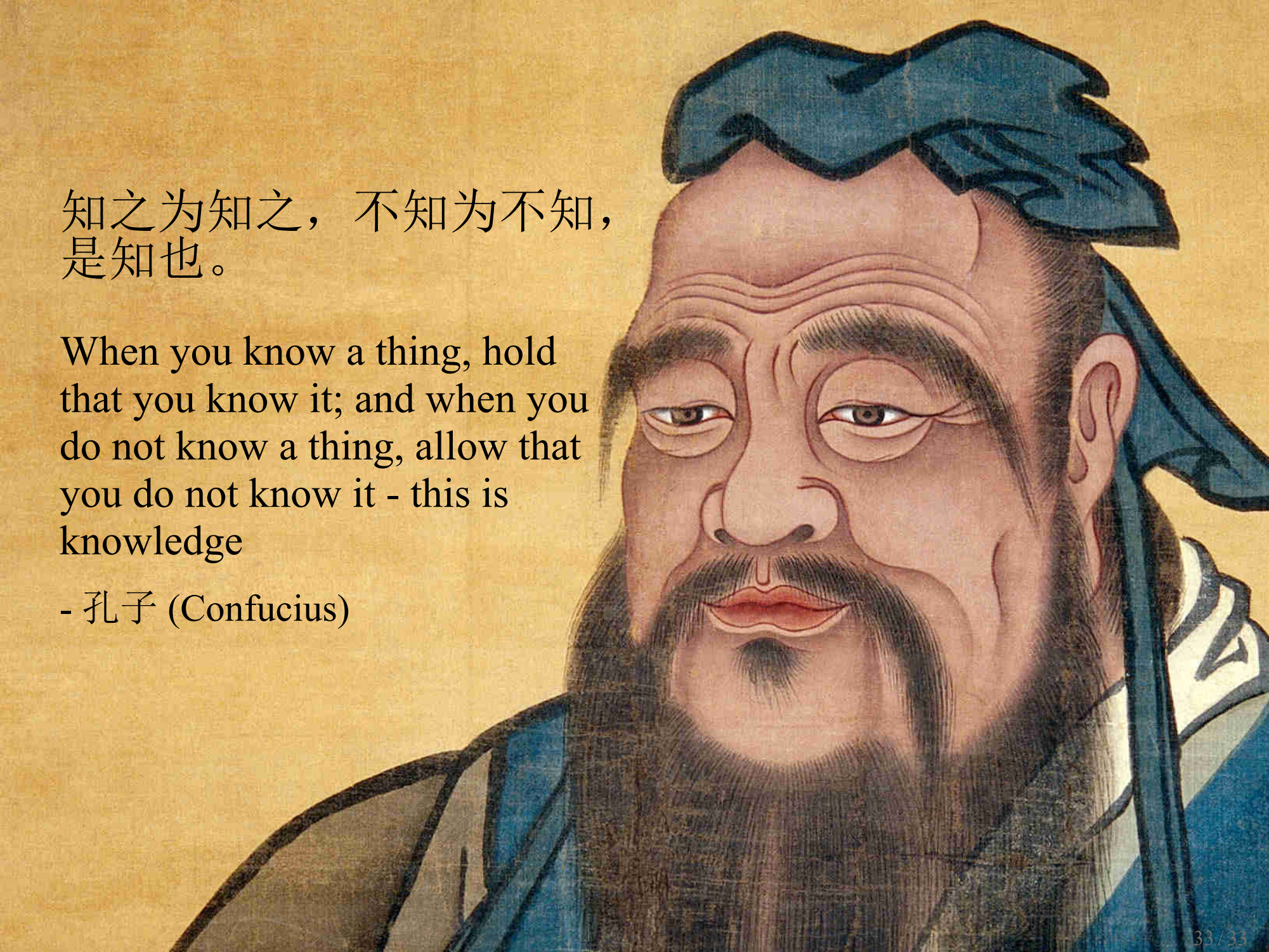
What We Have Achieved

- Tractable approach for confronting uncertainty
- Mechanism for inducing fluctuations in asset values: investors fear persistence in bad times and fear the lack of persistence in good times

A qualitatively similar mechanism will emerge with a recursive robust prior/posterior formulation and real-time learning.

Broader perspective

- Difficult to disentangle risk aversion from belief distortions
- Belief distortions are more compelling in environments in which uncertainty is complex
- Statistical tools provide valuable ways to assess environmental complexity
- Value to pushing beyond the risk model commonly embraced in economics and finance
- Ambiguity and model misspecification aversion induce nonlinearity dynamics into valuation



知之为知之，不知为不知，
是知也。

When you know a thing, hold that you know it; and when you do not know a thing, allow that you do not know it - this is knowledge

- 孔子 (Confucius)