Uncertainty and Valuation

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Recursive Valuation

- Use a recursive utility model (see Koopmans, Kreps & Porteus, Epstein & Zin, …) to highlight how uncertainty about future events affects asset valuation.
- Explore ways in which expectations and uncertainty about future growth rates influence risky claims to consumption.

Investigate how beliefs about the future are reflected in current-period assessments through continuation values. The *forward-looking* nature of the recursive utility model provides an additional channel through which *perceptions* about the future matter. (Bansal-Yaron and many others.)

Recursive Utility

Consider the aggregator specified in terms of C_t the current period consumption and V_t the continuation value:

$$V_t = \left[(C_t)^{1-\rho} + \exp(-\delta) \left[\mathcal{R}_t(V_{t+1}) \right]^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

where

$$\mathcal{R}_t \left(V_{t+1} \right) = \left(E \left[\left(V_{t+1} \right)^{1-\gamma} | \mathcal{F}_t \right] \right)^{\frac{1}{1-\gamma}}$$

adjusts the continuation value V_{t+1} for risk. $\frac{1}{\rho}$ is the elasticity of intertemporal substitution and δ is a subjective discount rate.

Stochastic Discount Factor

$$\frac{S_{t+1}}{S_t} = \exp(-\delta)\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left[\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}\right]^{\rho-\gamma}$$

- Continuation value enhances the impact of the perceptions about the future.
- Special case: Power utility sets $\rho = \gamma$.
- Multiply to compound over multiple periods.
- When $\rho = 1$

$$\left[\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}\right]^{1-\gamma} = \frac{(V_{t+1})^{1-\gamma}}{E\left[(V_{t+1})^{1-\gamma}|\mathcal{F}_t\right]}$$

has conditional expectation equal to unity. Equivalent interpretation as distorted beliefs.

Risk-Return Tradeoffs

Dynamic asset pricing through altering cash flow exposure to shocks.

- Study implication on the price today of changing the exposure tomorrow on a cash flow at some future date.
- Represent shock price elasticities by normalizing the exposure and studying the impact on the logarithms of the expected returns.
- Construct pricing counterpart to impulse response functions.

Impulse Problem

Ragnar Frisch (1933):

> *There are several alternative ways in which one may approach the impulse problem .... One way which I believe is particularly fruitful and promising is to study what would become of the solution of a determinate dynamic system if it were exposed to a stream of erratic shocks that constantly upsets the continuous evolution, and by so doing introduces into the system the energy necessary to maintain the swings.*

Irving Fisher (1930):

> *The manner in which risk operates upon time preference will differ, among other things, according to the particular periods in the future to which the risk applies.*

Elasticities

Counterparts to impulse response functions pertinent to valuation:

- shock-exposure elasticities
- shock-price elasticities

These are the ingredients to risk premia, and they have a term structure induced by the changes in the investment horizons.

Hansen-Scheinkman (*Finance and Stochastics*),
Borovička-Hansen-Hendricks-Scheinkman (*Journal of Financial Econometrics*), Hansen (Fisher-Schultz, *Econometrica*), Borovička and Hansen ((*Journal of Econometrics*),
Borovička-Hansen-Scheinkman (*Mathematical and Financial Economics*)

Quantitative Example (Bansal-Yaron)

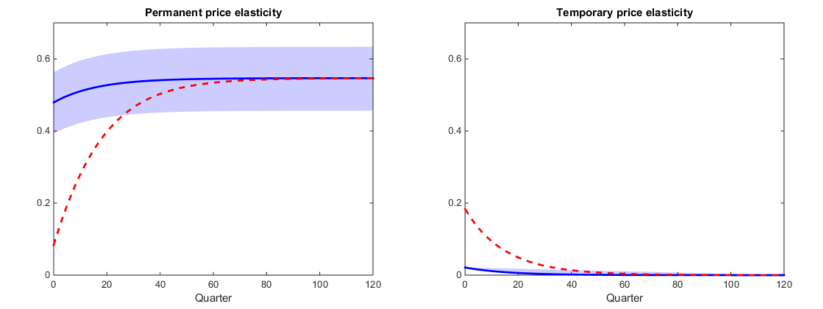
$$dZ_t^{[1]} = -.021 Z_t^{[1]} dt + \sqrt{Z_t^{[2]}} \begin{bmatrix} .031 & -.015 & 0 \end{bmatrix} dW_t$$

$$dZ_t^{[2]} = -013 \left(Z_t^{[2]} - 1 \right) dt + \sqrt{Z_t^{[2]}} \begin{bmatrix} 0 & 0 & -.038 \end{bmatrix} dW_t$$

$$dY_t = (.01)(.15 + Z_t^{[1]}) dt + (.01) \sqrt{Z_t^{[2]}} \begin{bmatrix} .34 & .7 & 0 \end{bmatrix} dW_t$$

- Y_t is the logarithm of consumption;
- $Z^{[1]}$ captures predictability in growth rates;
- $Z^{[2]}$ captures stochastic volatility;
- Components of dW_t:
  - Permanent shock;
  - Transitory shock;
  - Stochastic volatility shock.

Shock-Price Elasticities



Recursive utility and Power utility. Bands depict .1 and .9 deciles.

Success?

- The mechanism relies on endowing investors with knowledge of statistically subtle components of the macro time series. Where does this confidence come from?
- Imposes stochastic volatility exogenously.
- Imposes large risk aversion.

Risk Aversion or Subjective Belief Distortion

- Introduce a positive martingale process $\widetilde{M}$ with a unit expectation
- Form $\widetilde{S} = \frac{S}{\widetilde{M}}$
- Observe:

$$S = \widetilde{S} \times \widetilde{M}$$

- Use $\widetilde{M}$ to distort investor beliefs and use $\widetilde{S}$ as an alternative stochastic discount factor

Cannot distinguish belief distortions from stochastic discount factors without further restrictions!

Martingale Factorization Methods

Suppose that $\log S$ has stationary increments. Then S has three components

$$\frac{S_t}{S_0} = \exp(-\eta t) \left(\frac{M_t}{M_0} \right) \left(\frac{F_t}{F_0} \right)$$

where

- η long term yield on a discount bond;
- $\log M$ has stationary increments;
- $\log F$ is stationary.

References: Alvarez and Jermann (*Econometrica*), Hansen and Scheinkman (*Econometrica*) and Borovička-Hansen-Scheinkman (*Journal of Finance*)

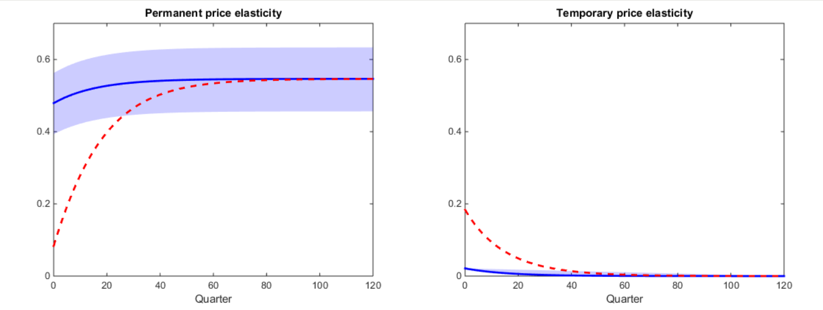
Martingale Factorization Methods

$$\frac{S_t}{S_0} = \exp(-\eta t)\left(\frac{M_t}{M_0}\right)\left(\frac{F_t}{F_0}\right)$$

- Probability associated with M is a long-term counterpart to a forward measure. Under this measure long-term risk prices for growth rate risk are degenerate.
- The positive martingale M embeds interesting economic considerations including stochastic growth and recursive utility.

Flat shock price elasticities for some shocks reflect a prominent martingale component to the stochastic discount factor.

Remember this Plot

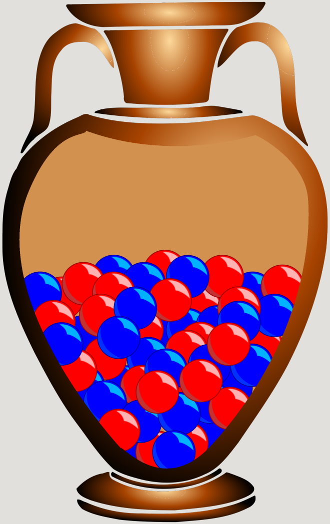


Shock price elasticities for Recursive utility and Power utility. Bands depict quartiles.

Uncertainty components

- risk -
  uncertainty within a model: uncertain outcomes with known probabilities
- ambiguity -
  uncertainty across models: unknown weights for alternative possible models
- misspecification -
  uncertainty about models: unknown flaws of approximating models

Uncertainty can be *risk*



50 Red Balls

50 Blue Balls

Uncertainty can be *ambiguity*

? Red Balls

? Blue Balls

Uncertainty can *change over time*

? Red Balls

? Blue Balls

What about Subjective Probability?

- De Finetti:
  "Subjectivists should feel obligated to recognize that any opinion (so much more the initial one) is only vaguely acceptable…So it is important not only to know the exact answer for an exactly specified initial problem, but what happens changing in a reasonable neighbourhood the assumed opinion."
- Savage:
  "No matter how neat modern operational definitions of personal probability may look, it is usually possible to determine the personal probabilities of events only very crudely."

Skepticism

Making Robustness Operational

- Explore a family of "posteriors/priors" used to weight models possibly relative to a benchmark specification. Dynamic learning plays a central role. (Robust Bayesian analysis)
- Explore a family of alternative potential models or a class of perturbations to a benchmark model subject to constraints or penalization. Future perturbations may not be tied to the past making learning about them impossible. (Control theory and statistical origins.)

Use the decision problem to target the member of the family that has the largest adverse utility consequences.

Worst-case Model of a Belief Distortion

- ▷ The analysis often yields a (constrained) worst-case probability.
- ▷ Apply the theory of two-person games. The decision maker optimizes taking as given the worst-case probability.
- ▷ Decentralize with worst-case probability.

Concerns about model misspecification look like belief distortions.

Formalization

- Construct a specification of preferences as in Hansen-Sargent(AER) and Maccheroni-Marinacci-Rustichini (*Econometrica*, *JET*)
- Relative entropy penalization gives a rationale for exponential tilting using the value function as the penalized worst-case probability:

$$\frac{\exp\left(-\frac{\log V_{t+1}}{\xi}\right)}{E\left[\exp\left(-\frac{\log V_{t+1}}{\xi}\right)|\mathcal{F}_t\right]} = \frac{(V_{t+1})^{1-\gamma}}{E\left[(V_{t+1})^{1-\gamma}|\mathcal{F}_t\right]}$$

  where V_{t+1} is the next-period continuation value and $1-\gamma = -\frac{1}{\xi}$.

No endogenous source for fluctuations in uncertainty prices.

Robustness concerns illustrated

- Initial model

$$dY_t = (.01)\left(\widehat{\alpha}_y + \widehat{\beta} Z_t\right) dt + (.01)\sigma_x \cdot dW_t$$

$$dZ_t = \widehat{\alpha}_z dt - \widehat{\kappa} Z_t dt + \sigma_z \cdot dW_t$$

- W a Brownian motion
- Think of Y as log consumption and use logarithmic utility
- Z generates "long-run risk" or growth-rate uncertainty

Basic idea

- A representative consumer has instantaneous utility $\log C_t = Y_t$
- Disguise drift distortions inside Brownian motions.
- Weak information on the parameters. Other distortions are allowable but the decision problem features these.
- A concern for robustness is reflected in the implied "risk"-return tradeoff over alternative investment horizons.

Potential Misspecification

- Change the evolution of W:

$$dW_t = U_t dt + dW_t^U$$

  where W^U is a Brownian motion and U_t is a history dependent drift distortion.
- Impose a quadratic penalty in the drift distortion. Link to likelihood ratios and statistical discrepancy.
- Worst-case drift distortion is constant. Increase uncertainty prices but in a manner that is *invariant* over time.

Enriching the Uncertainty Pricing Dynamics

Two approaches:

- Structural misspecification.
- Robust learning under misspecification - fragile beliefs.

I will report results based on the first approach. Results for the second can be found in Hansen and Sargent (QE,2010) and Hansen Ely Lecture (AER, 2007).

Family of Restricted Models

- parameters: $\alpha_y, \beta, \alpha_z, \kappa$
- evolution:

$$dY_t = .01\left(\alpha_y + \beta Z_t\right) dt + .01\sigma_y \cdot dW_t^R$$
$$dZ_t = \alpha_z dt - \kappa Z_t dt + \sigma_z \cdot dW_t^R$$

- Construct drift distortion for the Brownian motion $dW_t = R_t dt + dW_t^R$ where $R_t = \eta(Z_t) \equiv \eta_0 + \eta_1 Z_t$ and where

$$\sigma = \begin{bmatrix} (\sigma_y)' \\ (\sigma_z)' \end{bmatrix},$$

and

$$\sigma\eta_0 = \begin{bmatrix} \alpha_y - \hat{\alpha}_y \\ \alpha_z - \hat{\alpha}_z \end{bmatrix} \quad \sigma\eta_1 = \begin{bmatrix} \beta - \hat{\beta} \\ \hat{\kappa} - \kappa \end{bmatrix}$$

- Impose local restriction (see Chen-Epstein, Econometrica).

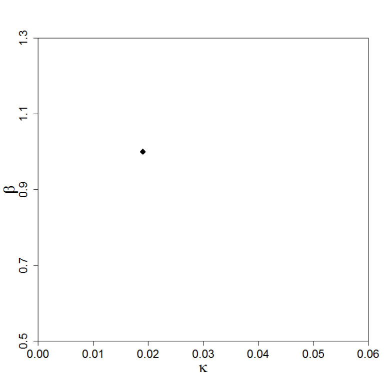
Uncertainty and Financial Markets

Bear Bull Rumble, Adrian deRooy

Slope Uncertainty

$$dY_t = .01\left(\alpha_y dt + \beta Z_t dt + \sigma_y \cdot dW_t\right) \qquad \text{macro evolution}$$

$$dZ_t = \alpha_z dt - \kappa Z_t dt + \sigma_z \cdot dW_t \qquad \text{growth evolution}$$



Sets of parameter values (β, κ) constrained by relative entropy.

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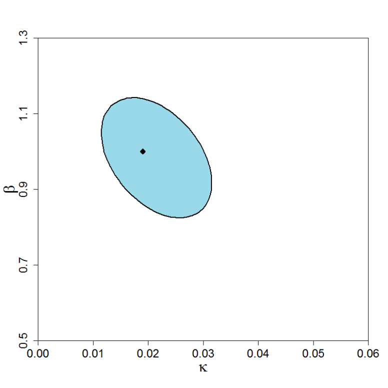

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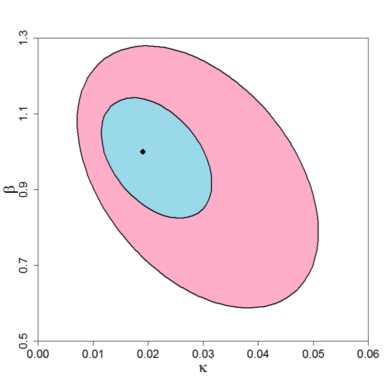


Sets of parameter values (β, κ) constrained by relative entropy.

Misspecified Dynamics

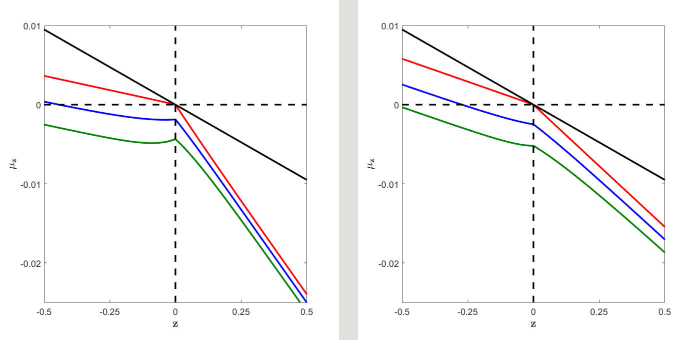
- Introduce unstructured drift distortions U_t:

$$dW_t = U_t dt + dW_t^U$$

- impose a quadratic penalty $|U_t - R_t|^2$ where R_t is one of the possible modeled drift distortions
- minimize over the restricted family of R_t's

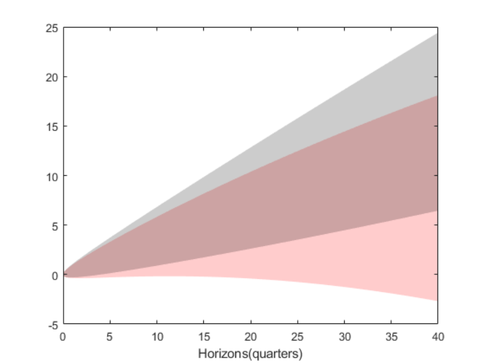
Special case of variational preferences and extends Hansen-Sargent (AER).

Local Growth Rate Uncertainty



Growth rate drifts. Left panel: larger structured entropy. Right panel: smaller structured entropy. **Black**: baseline model; **red**: worst-case structured model; **blue**: $\mathsf{q}_{u,s} = .1$; and **green**: $\mathsf{q}_{u,s} = .2$. Reference: Hansen-Sargent, *Tenuous Beliefs and the Price of Uncertainty*.

Tilting Probabilities



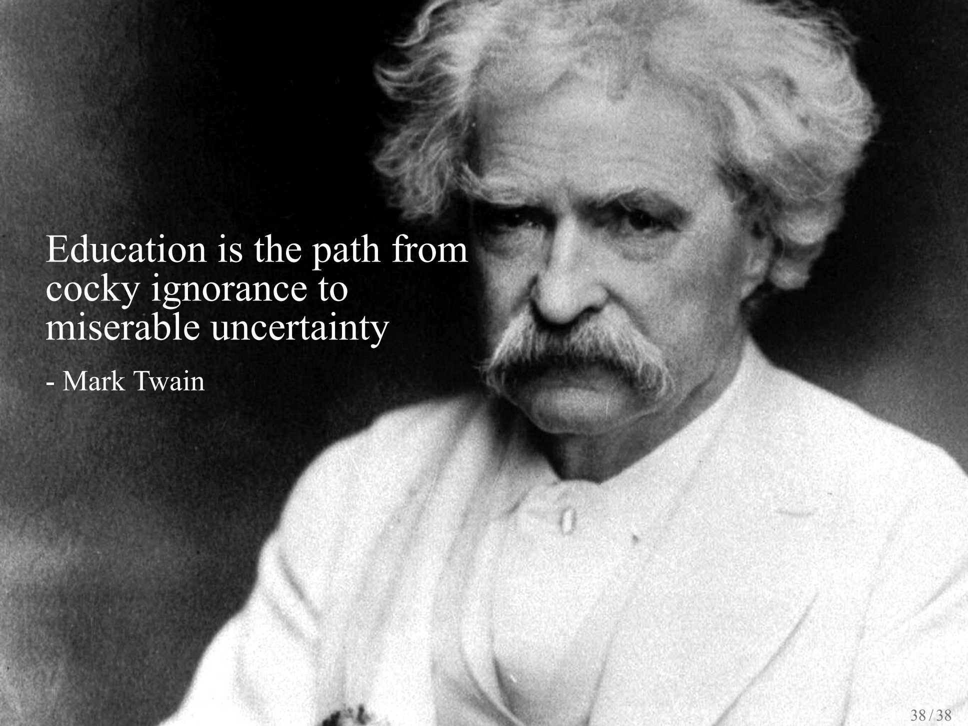
Distribution of $Y_t - Y_0$ under the baseline model and worst-case model. The **black** solid line depicts the baseline median and gray shaded region are includes .1- .9 deciles. The red dashed line is the median under the worst-case model and the red shaded region includes the .1-.9 deciles.

What We Have Achieved

- tractable approach for confronting uncertainty
- mechanism for inducing fluctuations in asset values
- investors fear persistence in bad times and fear the lack of persistence in good times

Broader Perspective

- difficult to disentangle risk aversion from belief distortions
- belief distortions are more compelling in environments in which uncertainty is complex
- statistical tools provide valuable ways to assess environmental complexity
- value to pushing beyond the risk model commonly embraced in economics and finance


Education is the path from cocky ignorance to miserable uncertainty
- Mark Twain