Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns

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This paper studies the time-series behavior of asset returns and aggregate consumption. Using a representative consumer model and imposing restrictions on preferences and the joint distribution of consumption and returns, we deduce a restricted log-linear time-series representation. Preference parameters for the representative agent are estimated and the implied restrictions are tested using postwar data.

I. Introduction

In the asset pricing models of Rubinstein (1976*b*), Lucas (1978), Breeden (1979), and Brock (1982), among others, agents effect their consumption plans by trading shares of ownership of firms in a competitive stock market. An implication of this trading is that the serial correlation properties of stock returns are intimately related to the stochastic properties of consumption and the degree of risk aversion of investors. The purposes of this paper are to characterize explicitly

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the restrictions on the joint distribution of asset returns and consumption implied by a class of general equilibrium asset pricing models and to obtain maximum likelihood estimates of the parameters describing preferences and the stochastic consumption process.

The motivation for this analysis derives from two considerations. First, in general equilibrium models of stock price behavior with risk-neutral agents (i.e., linear utility), share prices will be set so that the expected return on each asset is constant. Thus, asset returns will be serially uncorrelated and, in particular, past values of consumption will be uncorrelated with current-period asset returns. LeRoy and Porter (1981) and Shiller (1981) have recently conducted tests of the linear present-value formula for stock prices, implied by this result, and in both studies the model was rejected. As Grossman and Shiller (1981) have emphasized, these rejections suggest that agents do consider consumption risk when making portfolio decisions. Second, if agents are risk averse, then the temporal covariance structure of consumption and asset returns will be nontrivial, except under very strong restrictions on the underlying production technology (see, e.g., Rubinstein [1976*b*], Johnsen [1978], and Sec. II below). It is this temporal covariation that we attempt to characterize here.

The framework for this analysis is a production-exchange economy of identical agents who choose consumption and investment plans so as to maximize the expected value of a time-additive von Neumann–Morgenstern utility function. In order to derive the restrictions on the joint distribution of consumption and stock returns implied by this optimizing behavior, it is necessary to specify a distribution function and to parameterize preferences. The joint distribution of consumption and returns is assumed to be lognormal, and preferences are assumed to exhibit constant relative risk aversion (CRRA). This particular form of utility was chosen in part because of its preeminent role in many previous theoretical studies of asset pricing (e.g., Merton 1973; Rubinstein 1976*a*). In addition, the assumptions of CRRA utility and lognormality together lead to an empirically tractable, closed-form characterization of the restrictions implied by the model.[1]

More precisely, these assumptions lead to a restricted linear time-series representation of the logarithms of consumption and asset returns. The restrictions imply that the predictable components of the

[1] A similar interplay among the CRRA utility function and lognormal returns was exploited by Merton (1973, 1980), Rubinstein (1976*b*), Breeden (1977), and Grossman and Shiller (1981), among others, to obtain closed-form solutions to their models. Breeden and Litzenberger (1978) derive a version of the CAPM for a model with CRRA utility and lognormal returns and consumption.

logarithms of asset returns are proportional to the predictable component of the change in the logarithm of consumption, with the proportionality factor being minus the coefficient of relative risk aversion. Maximum likelihood estimates of the coefficient of relative risk aversion, the subjective discount factor, and the parameters that describe the temporal evolution of consumption are obtained using this closed-form characterization of the restrictions. The model is estimated for returns on stocks listed on the New York Stock Exchange and for returns on Treasury bills, using monthly data for the period 1959:2 through 1978:12. Then likelihood ratio tests of the joint hypothesis underlying the model are conducted.

The remainder of this paper is organized as follows. In Section II the model is described and the implied time-series representation for consumption and returns is derived. In Section III, maximum likelihood estimation is discussed and estimates of the parameters are presented. Concluding remarks are presented in Section IV.

II. The Model of Stock Market Returns

Consider a single-good economy of identical consumers, whose utility functions are of the CRRA type:

$$U(c_t) = c_t^{\gamma}/\gamma; \quad \gamma < 1, \tag{1}$$

where c_t is aggregate real per capita consumption and $U(\cdot)$ is the period utility function. The representative consumer in this economy is assumed to choose a stochastic consumption plan so as to maximize the expected value of his time-additive utility function,

$$E_0\left[\sum_{t=0}^{\infty} \beta^t U(c_t)\right], \quad 0 < \beta < 1. \tag{2}$$

In (2), β is a discount factor and $U(\cdot)$ is given by (1). The mathematical expectation $E_t(\cdot)$ is conditioned on information available to agents at time t, I_t. Current and past values of real consumption and asset returns are assumed to be included in I_t.

Consumers substitute present for future consumption by trading the ownership rights of N financial and capital assets. These assets include default-free, multiperiod bonds that agents issue for the purpose of borrowing or lending among themselves and shares of ownership of firms in the economy. If firms rent capital from consumers, as in Brock's (1982) model, then the stocks of capital leased to the firms by the representative consumer will also be included among the traded assets. Let $\mathbf{w}_t$ denote the holdings of the N assets at the date t, $\mathbf{q}_t$ denote the vector of prices of the N assets in $\mathbf{w}_t$ net of any distributions, and $\mathbf{q}_t^*$ denote the vector of values of these distributions during period t. Then a feasible consumption and investment plan $\{c_t, \mathbf{w}_t\}$ must satisfy the sequence of budget constraints,

$$c_t + \mathbf{q}_t \cdot \mathbf{w}_{t+1} \leq (\mathbf{q}_t + \mathbf{q}_t^*) \cdot \mathbf{w}_t + y_t, \tag{3}$$

where y_t is the level of (real) labor income at date t.[2]

The first-order necessary conditions for the maximization of (2) subject to (3), that involve the equilibrium prices of the n assets, are (Lucas 1978; Brock 1982)

$$U'(c_t) = \beta E_t[U'(c_{t+1})r_{it+1}]; \quad i = 1, \ldots, N, \tag{4}$$

where r_{it+1} is the return on the ith asset expressed in units of the consumption good. Substituting (1) into (4) and rearranging gives

$$E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{\alpha} r_{it+1}\right] = 1; \quad i = 1, \ldots, N, \tag{5}$$

with $\alpha \equiv \gamma - 1$. Breeden (1979) has derived an intertemporal capital asset pricing representation in a continuous-time environment. In his representation, expected excess returns on risky assets are linked to covariances of aggregate consumption and returns. Grossman and Shiller (1981) have shown how to obtain an analogous representation for the discrete time model studied here. Their representations are useful for studying the riskiness of a cross section of asset returns. The focus of this paper is instead on the link between forecastable movements in consumption and forecastable movements in asset returns. Accordingly, we proceed to derive a relation among these forecastable components implied by (5).

For the analysis of (5) that follows, it is not necessary to examine explicitly firms' production decisions, since it is not our goal to solve for an explicit representation of equilibrium prices in terms of the underlying shocks to technology. By assuming that the joint distribution of consumption and returns is lognormal, we are implicitly imposing restrictions on the production technology, however. As in many previous theoretical and empirical studies of asset pricing (see n. 1 above), we leave unspecified the exact nature of these restrictions. A formal justification of the assumption of lognormality can be provided for some economic environments for which closed-form equilibrium pricing functions have been derived. Such a justification has not been provided at the level of generality at which our empirical analysis is conducted, however. We adopt our general representation

[2] The inclusion of y_t does not affect our analysis if labor is supplied inelastically. Alternatively, we can introduce a period t labor supply variable, L_t, into the specification of U and let $U(c_t, L_t) = U_1(c_t) - U_2(L_t)$, where L_t is a choice variable of the consumer. For this case, $y_t = L_tW_t$, where W_t is the real wage rate at date t.

to accommodate a rich temporal covariance structure which might emerge when the investment environment faced by firms is more complicated than the environments in the models of Lucas (1978) and Brock (1982) (e.g., serially correlated production shocks, costly adjustment in altering capital stocks, and gestation lags in producing new capital).

From (5) and the accompanying assumptions, a restricted linear time-series representation of the logarithms of consumption and asset returns can be derived. Suppose that observations on the first n of the N assets traded by economic agents are to be used in the econometric analysis. Let $x_t \equiv c_t/c_{t-1}$ and $u_{it} \equiv x_t^{\alpha} r_{it}$, $i = 1, \ldots, n$. Then (5) can be rewritten as

$$E_{t-1}(u_{it}) = 1/\beta, \quad i = 1, \ldots, n. \tag{6}$$

Next, let $X_t \equiv \log x_t$, $R_{it} \equiv \log r_{it}$, $\mathbf{Y}_t = (X_t, R_{1t}, \ldots, R_{nt})'$, $U_{it} = \log u_{it}$ $(i = 1, \ldots, n)$, and ψ_{t-1} denote the information set $\{\mathbf{Y}_{t-s}\colon s \geq 1\}$. Further, assume that $\{\mathbf{Y}_t\}$ is a stationary Gaussian process. This distributional assumption implies that the distribution of U_{it} conditional on ψ_{t-1} is normal with a constant variance σ_i^2 and a mean μ_{it-1} that is a linear function of past observations on $\mathbf{Y}_t$. Hence,

$$E(u_{it}|\psi_{t-1}) = \exp\,[\mu_{it-1} + (\sigma_i^2/2)]. \tag{7}$$

Since $\psi_{t-1} \subset I_{t-1}$, we can take expectations of both sides of (6) conditional on ψ_{t-1} to obtain

$$E(u_{it}|\psi_{t-1}) = 1/\beta. \tag{8}$$

Equating the right-hand sides of equations (7) and (8) and solving for μ_{it-1} yields $\mu_{it-1} = -\log\beta - (\sigma_i^2/2)$. Define

$$V_{it} \equiv U_{it} - \mu_{it-1} = \alpha X_t + R_{it} + \log\beta + (\sigma_i^2/2), \quad i = 1, \ldots, n. \tag{9}$$

Then, $E(V_{it}|\psi_{t-1}) = 0$ and

$$E(R_{it}|\psi_{t-1}) = -\alpha E(X_t|\psi_{t-1}) - \log\beta - (\sigma_i^2/2), \quad i = 1, \ldots, n. \tag{10}$$

Equations (9) and (10) summarize the relationships among serial correlation of consumption, the level of risk aversion, and serial correlation of asset returns implied by the first-order conditions (5). Risk neutrality, for example, corresponds to the case of $\alpha = 0$, which implies that R_{it} is equal to a constant plus the serially uncorrelated error V_{it} and hence that R_{it} is serially uncorrelated, $i = 1, \ldots, n$. Alternatively, if $\alpha = -1$, then agents have logarithmic utility functions. In this case, $R_{it} - X_t = -\log\beta - (\sigma_i^2/2) + V_{it}$. Thus, the slope coefficients in the projections of R_{it} and X_t onto a subset ψ_{t-1} of I_{t-1} must be the same, and this equality must hold for the returns on all assets. More generally, (10) implies that (ignoring constant terms) the coefficients in the projection of R_{it} onto ψ_{t-1} are equal to the coefficients in the projection of X_t onto ψ_{t-1} multiplied by $-\alpha$.

To translate these observations into statements about the predictability of asset returns, it is useful to derive an expression for the coefficient of determination (R_i^2) from the projection of R_{it} onto ψ_{t-1} implied by (10). By definition,

$$R_i^2 = \frac{\mathrm{var}\,[E(R_{it}|\psi_{t-1})]}{\mathrm{var}\,(R_{it}|\psi_{t-1}) + \mathrm{var}\,[E(R_{it}|\psi_{t-1})]}, \tag{11}$$

where var is the variance operator. From (10) it follows that the variances of the predictable components of $\log r_{it}$ and $\log (c_t/c_{t-1})$ are related by the expression:

$$\mathrm{var}\,[E(R_{it}|\psi_{t-1})] = \alpha^2\,\mathrm{var}\,[E(X_t|\psi_{t-1})]. \tag{12}$$

Substituting (12) into (11) gives

$$R_i^2 = \frac{\alpha^2\,\mathrm{var}\,[E(X_t|\psi_{t-1})]}{\mathrm{var}\,(R_{it}|\psi_{t-1}) + \alpha^2\,\mathrm{var}\,[E(X_t|\psi_{t-1})]}. \tag{13}$$

From (13) it follows that a necessary condition for asset returns to have predictable components is that agents be risk averse ($\alpha \neq 0$).

Risk aversion is not a sufficient condition for predictability, however. For the special case in which the projection $E(X_t|\psi_{t-1})$ is constant, the R_i^2's are equal to zero or, equivalently, the projections of the R_{it} onto ψ_{t-1} are constants. This implication of our model is consistent with the conclusion of Rubinstein (1976*b*) that asset returns will be serially uncorrelated when consumption follows a logarithmic random walk and agents have CRRA preferences. When there are nontrivial predictable components in X_t and $\alpha \neq 0$, then real asset returns will also have predictable components.

The assumption that the vector process $\{\mathbf{Y}_t\}$ is stationary and Gaussian implies that the conditional expectations in (10) have linear, time-invariant representations and that the conditional variances are constant (a fact that we have exploited above). Thus, the movements in the conditional distributions of the logarithms of consumption and asset returns are completely summarized by movements in the conditional means. This distributional specification leads to a very convenient representation of the intertemporal behavior of consumption and asset returns for the purposes of empirical analyses. Once the projection $E(X_t|\psi_{t-1})$ is parameterized as a linear function of past values of $\mathbf{Y}_t$, the free parameters of (10) can be estimated by the method of maximum likelihood and the overidentifying restrictions can be tested using the likelihood ratio statistic. Since our characterization of the overidentifying restrictions relies on an assumption about the joint distribution of consumption and returns, rejection of these restrictions may result from misspecifying that distribution rather than from the empirical failure of the time-additive CRRA preference form of the asset pricing model.

Other authors have studied this asset pricing model by relying on the same distributional assumption as that employed here. Grossman and Shiller (1981) have shown how to identify preference parameters under a joint lognormality assumption on consumption and returns. They abstain from studying the intertemporal correlations of these variables and express the estimators of their preference parameters as functions of the first and second unconditional moment of two returns. Hall (1981) has independently adopted an approach that is very similar to the one employed here to estimate α for different data sets.[3] Neither of these studies considers tests of overidentifying restrictions.

III. Maximum Likelihood Estimates of the Parameters

To proceed with estimation, we assume that

$$E(X_t|\psi_{t-1}) = \mathbf{a}(L)'\mathbf{Y}_{t-1} + \mu_x, \tag{14}$$

where $\mathbf{a}(L)$ is an $n + 1$ dimensional vector of finite order polynomials in the lag operator L. Combining equations (14) and (9) gives

$$A_0\mathbf{Y}_t = A_1(L)\mathbf{Y}_{t-1} + \boldsymbol{\mu} + \mathbf{V}_t, \tag{15}$$

where $\mathbf{V}_t = (W_t, V_{1t}, \ldots, V_{nt})'$ and $W_t \equiv X_t - E(X_t|\psi_{t-1})$. The matrix A_0 is given by

$$A_0 = \begin{bmatrix} 1 & 0 \\ \boldsymbol{\alpha} & I \end{bmatrix},$$

with $\boldsymbol{\alpha} = (\alpha, \alpha, \ldots, \alpha)'$ and I an $n \times n$ identity matrix; the matrix lag polynomial $A_1(L)$ is given in partitioned form by

$$A_1(L) = \begin{bmatrix} \mathbf{a}(L)' \\ 0 \end{bmatrix};$$

[3] There are two differences in the estimation strategy employed by Hall (1981). First, he assumes that economic agents do not know the true parameter values in the forecasting equation for asset returns. Instead, they use Bayesian updating formulas as they accumulate new information over time about these parameters. Second, he expands the vector $\mathbf{Y}_t$ to include variables other than asset returns and consumption.

and the vector of constants $\boldsymbol{\mu}$ is given by

$$\boldsymbol{\mu} = [\mu_x, \log \beta + (\sigma_1^2/2), \ldots, \log \beta + (\sigma_n^2/2)]'.$$

From equation (9) it follows that $\{(W_{t-s}, V_{1t-s}, \ldots, V_{nt-s}); s \geq 0\}$ spans the space ψ_t. Hence, the autoregressive representation of $\mathbf{Y}_t$ is obtained by premultiplying both sides of (15) by A_0^{-1}.[4]

Now let $\boldsymbol{\theta}$ denote the vector of unknown parameters containing α, β, μ_x, the parameters of $\mathbf{a}(L)$, and the elements of the covariance matrix of $\mathbf{V}_t$, denoted by Σ. It is assumed that Σ is nonsingular. Suppose that T observations on $\mathbf{Y}$ are available for estimation of $\boldsymbol{\theta}$. Then, in view of the relation (15), the joint density function of the sample, conditioned on the initial values of the variables, is given by

$$f(\mathbf{Y}_1, \ldots, \mathbf{Y}_T; \boldsymbol{\theta}) = (2\pi)^{-(n+1)T/2}|\Sigma|^{-T/2} \tag{16}$$
$$\exp\left\{-(\tfrac{1}{2}) \sum_{t=1}^{T} [A_0\mathbf{Y}_t - A_1(L)\mathbf{Y}_{t-1} - \boldsymbol{\mu}]'\Sigma^{-1}[A_0\mathbf{Y}_t - A_1(L)\mathbf{Y}_{t-1} - \boldsymbol{\mu}]\right\}.$$

Note that (16) is also the joint density function of $(\mathbf{V}_1, \ldots, \mathbf{V}_T)$, since the Jacobian of the transformation A_0^{-1} that transforms (15) into the autoregressive representation is unity. The logarithm of the conditional likelihood function (16) is, up to a constant term,

$$L(\mathbf{Y}_1, \ldots, \mathbf{Y}_T; \boldsymbol{\theta}) =$$
$$-(T/2) \log |\Sigma| - (\tfrac{1}{2}) \sum_{t=1}^{T} [A_0\mathbf{Y}_t - A_1(L)\mathbf{Y}_{t-1} - \boldsymbol{\mu}]'\Sigma^{-1} \tag{17}$$
$$\times [A_0\mathbf{Y}_t - A_1(L)\mathbf{Y}_{t-1} - \boldsymbol{\mu}].$$

The maximum likelihood (ML) estimate of $\boldsymbol{\theta}$ is obtained by maximizing (17). Unfortunately, unless $n = 1$, the conditional log-likelihood function cannot be concentrated, because $\boldsymbol{\mu}$ is a function of the parameters in Σ.

Estimates were obtained using monthly data for the period 1959:2 through 1978:12. The monthly, seasonally adjusted real consumption series, dating back to January 1959, were obtained from the CITIBASE data tape. The observations of these series were divided by the monthly estimates of population published by the Bureau of the Census to get per capita values. The ML estimates are reported for two alternative measures of consumption: nondurables plus services (NDS) and nondurables (ND). Hall (1978) and Grossman and Shiller (1981) used the former measure, while Flavin (1981) and Hall

[4] The following estimation and testing procedures can be modified to accommodate vector autoregressive moving average representations of $\mathbf{Y}_t$, including representations with unit roots in the moving average polynomial that might be induced by differencing.

(1981) used the latter measure of consumption. We maintained the usual practice of excluding durables from measured consumption, due to the difficulty of imputing a service flow to the stock of durables.

Several monthly asset return series were studied. Return series for two levels of aggregation across common stocks were considered: an average return on all stocks listed on the New York Stock Exchange and returns on individual members of the Dow Jones Industrials. In addition to stock returns, we considered the 1-month return on Treasury bill yields. The stock return data were obtained from the Center for Research in Security Prices (CRSP) tapes, and the Treasury bill data were obtained from Ibbotson and Sinquefield (1979). Nominal returns were converted to real returns, which appear in (5), with the implicit price deflator corresponding to the measure of consumption.

Each combination of a measure of consumption and an asset return potentially corresponds to a different underlying model of economic behavior. A sufficient condition for the restrictions in equation (10) to hold for a measure of a component of aggregate consumption is that preferences be separable in consumption. Specifically, suppose that $c_{1t} + c_{2t} = c_t$ and that the function U is given by

$$U(c_{1t}, c_{2t}) = (c_{1t}^{\gamma}/\gamma) + U_2(c_{2t}). \tag{18}$$

Then it is appropriate to test the model with c_1 used as the measure of consumption. A separability assumption similar to that underlying (18) is implicit in all of the previous empirical studies of consumption and asset returns that use only a component of aggregate consumption. By estimating models with nondurables and nondurables plus services, we are implicitly considering two different assumptions about the separability of preferences. Similarly, the choice among stock returns amounts to choosing among different models of the return generating process. All of the returns must satisfy a condition analogous to (5), if (5) holds for individual stocks. However, both the individual and aggregated return series will not in general be lognormally distributed.

To our knowledge, this is the first empirical study of consumption and asset returns that uses monthly data.[5] The variable c_t in (5) represents the level of consumption over the period of time between decisions of economic agents. In using monthly consumption data, we assume that the representative agent makes consumption decisions at monthly time intervals. Further, we assume that the representative agent knows the return measured from the beginning of the month

[5] Breeden (1980) and Hall (1981) used quarterly data and Grossman and Shiller (1981) used annual data, e.g.

until the end of the month in deciding how much to consume during that month. If the appropriate decision period is shorter than 1 month or if the timing of the consumption decision is incorrectly aligned with the available information on asset returns, then our statistical model is misspecified even if the underlying economic model is correct. Of course, if the decision period is shorter than 1 month, then measurement errors will also be present (and indeed may be much larger) in studies using quarterly or annual time averages of consumption.[6] These potential measurement and timing problems are avoided by some of the tests that we have conducted.

Consider first the results for the value-weighted return on stocks listed on the New York Stock Exchange, which are summarized in table 1. Models were estimated with two, four, and six lags in the lag polynomial $L\mathbf{a}(L)$; NLAG = 2, 4, 6. For each model, the estimated values of α and β, the coefficients of determination from the unrestricted vector autoregressions of consumption (R_c^2) and returns (R_R^2), and the χ^2 statistic, $\chi^2(df)$, for testing the overidentifying restrictions implied by (10) are presented. None of the test statistics has probability values that are larger than .90 and there is a tendency for the probability values to decline with increases in NLAG.

An explanation of this inverse relationship among the probability values and the choice of NLAG can be obtained from the unrestricted autoregressive representation. Estimates of the autoregressive coefficients for the six-lag model with consumption measured as nondurables plus services (model 6) are presented in table 2. These results suggest that values of R and X beyond the second lag are not very useful in forecasting consumption. Consequently, as df increases with NLAG, there are relatively smaller increases in the χ^2 statistics.

Although the estimators of α and β are consistent even if NLAG is misspecified, the point estimates of α are quite sensitive to the choice of NLAG. At the same time, the corresponding standard errors are relatively large. Evidently, precise estimates of α cannot be obtained with the data set and choice of information set used here. Nevertheless, all of the estimated values of α displayed in table 1 are economically plausible except for model 4, which yields an estimate in the nonconcave region of the parameter space. The estimates of α for models 3 and 6 (NLAG = 6) imply a slightly larger degree of risk aversion than is implied by a logarithmic period utility function (α =

[6] In constructing both monthly and quarterly consumption numbers, the Department of Commerce relies on a substantial amount of interpolation between yearly benchmark observations. Monthly aggregate consumption is estimated using monthly data on important subsets of consumption, indicators of various components of aggregate consumption, and yearly benchmarks for proportional breakdowns of consumption into various categories. For a more detailed description, see Byrnes et al. (1979).

TABLE 1

Summary of Maximum Likelihood Results for Value-Weighted Aggregate Return

Model	$\hat{\alpha}$*	$\hat{\beta}$*	CONS	NLAG	R_c^2	R_R^2	χ^2†	df
1	−.325 (.828)	.9976 (.0032)	ND	2	.187	.020	6.088 (.893)	3
2	−.831 (.746)	.9985 (.0030)	ND	4	.206	.040	8.426 (.703)	7
3	−1.25 (.647)	.9993 (.0030)	ND	6	.246	.056	10.622 (.524)	11
4	.359 (1.880)	.9965 (.0048)	NDS	2	.119	.012	4.980 (.827)	3
5	−.264 (1.835)	.9979 (.0045)	NDS	4	.128	.028	6.687 (.538)	7
6	−1.509 (1.571)	1.0007 (.0042)	NDS	6	.149	.048	10.932 (.551)	11
7‡	−2.721 (3.187)	.9957 (.0627)	ND	2	.201	.074	5.130 (.838)	3
8‡	−2.671 (9.306)	.9989 (.1442)	ND	4	.445	.111	10.030 (.813)	7

* Standard errors are indicated in parentheses.
† Probability values are indicated in parentheses.
‡ Estimated using quarterly data: 1954:4–1978:4.

TABLE 2

Maximum Likelihood Estimates for Model 6
(C = Nondurables + Services, NLAG = 6)

Restricted model ($\hat{\alpha} = -1.5088$ [1.571]; $\hat{\beta} = 1.007$ [.0042]):

$$
\begin{array}{llllll}
X = & .0027 & -\ .334X & -\ .147X_{-2} & +\ .074X_{-3} & +\ .086X_{-4} \\
 & (.0009) & (.071) & (.084) & (.077) & (.074) \\
 & -\ .025X_{-5} & +\ .006X_{-6} & +\ .002R_{-1} & +\ .012R_{-2} & \\
 & (.081) & (.069) & (.007) & (.008) & \\
 & +\ .012R_{-3} & +\ .004R_{-4} & +\ .003R_{-5} & -\ .014R_{-6} & \\
 & (.007) & (.007) & (.008) & (.008) &
\end{array}
$$

Unrestricted model:

$$
\begin{array}{llllll}
R = & .0065 & -\ .502X_{-1} & +\ .033X_{-2} & -\ .468X_{-3} & -\ .623X_{-4} \\
 & (.005) & (.683) & (.717) & (.722) & (.717) \\
 & -\ .827X_{-5} & -\ .239X_{-6} & +\ .098R_{-1} & -\ .056R_{-2} & \\
 & (.705) & (.667) & (.069) & (.069) & \\
 & +\ .114R_{-3} & +\ .073R_{-4} & +\ .076R_{-5} & -\ .114R_{-6}; & R^2 = .048 \\
 & (.071) & (.071) & (.071) & (.071) & \\
X = & .0028 & -\ .334X_{-1} & -\ .144X_{-2} & +\ .069X_{-3} & +\ .078X_{-4} \\
 & (.0005) & (.069) & (.072) & (.073) & (.072) \\
 & -\ .033 & +\ .003X_{-6} & +\ .003R_{-1} & +\ .011R_{-2} & \\
 & (.071) & (.067) & (.007) & (.007) & \\
 & +\ .012R_{-3} & +\ .004R_{-4} & +\ .004R_{-5} & -\ .015R_{-6}; & R^2 = .149 \\
 & (.007) & (.007) & (.007) & (.007) &
\end{array}
$$

Note.—Standard errors are indicated in parentheses.

-1). The estimated values of β are less than, but close to, unity as expected.

The R^2's from the unrestricted, bivariate autoregressions are also reported in table 1. The R_c^2's for models 3 and 6 (NLAG = 6) are .246 and .149, respectively. It is clear that monthly differences in the logarithms of consumption (NDS and ND) are serially correlated. From expression (13) we know that the model implies that stock returns will also be serially correlated, but R_R^2's may be much smaller than R_c^2 if var $(R_{it}|\psi_{t-1})$ is large relative to the numerator of (13). Indeed, the R_R^2's displayed in table 1 are relatively small, with the R_R^2's for the models using ND as a measure of consumption larger than those for the models using NDS.

For comparison, we have estimated two quarterly models of asset returns and consumption (models 7 and 8 in table 1). The measure of consumption for both models was calculated by summing the monthly observations of nondurables over quarters. The point estimates for the coefficient of relative risk aversion are larger than those obtained for the monthly models, but again the estimates are not very precise, especially for NLAG = 4.

The discussion to this point has focused on the behavior of an aggregate average stock return. If a set of n returns on individual stocks is jointly lognormally distributed with consumption, then the restrictions in (10) can be tested using these returns. We estimated the free parameters in $\boldsymbol{\theta}$ for a model including the returns on the stocks of three Dow Jones Industrials: American Brand, Exxon, and IBM. The nominal individual returns were obtained from the CRSP tapes and were converted to real returns in the same manner that the value-weighted return was converted. The assumption that the individual return series are lognormally distributed is, of course, inconsistent with the assumption that the aggregate return series is lognormal. We are, therefore, testing different models than those considered above. The likelihood ratio tests for both measures of consumption are reported in table 3 for the period February 1959 through December 1978. The three stock models are rejected by the data at essentially any significance level for both values of NLAG. Notice also that the estimated values of $|\alpha|$ increase with NLAG and are larger than unity when NLAG = 4.

The restrictions on asset returns implied by equation (10) should also hold for returns on bonds. To gain some insight into whether stocks and bonds yield qualitatively similar results, we estimated a model for the return of 1-month Treasury bills. The results are displayed in table 4. The estimates of α and β are quantitatively similar to the estimates obtained using stock return data. Note also that, for each value of NLAG, R_R^2 is much larger for the Treasury bill data

TABLE 3

LIKELIHOOD RATIO TESTS FOR THE MODELS OF INDIVIDUAL DOW JONES RETURNS (1959:2–1978:12)

	$\hat{\alpha}$	$\hat{\beta}$	χ^2	df	Probability
C = Nondurables:					
NLAG = 2	−.466	.995	310.3	25	1.000
NLAG = 4	−1.738	.997	334.7	49	1.000
C = Nondurables plus services:					
NLAG = 2	−.507	.995	238.5	25	1.000
NLAG = 4	−4.106	1.003	393.7	49	1.000

than the stock return data. This finding alone is evidence neither for nor against the model, even though the corresponding values of $\hat{\alpha}$ and R_c^2 are similar across tables 1 and 4. As noted above, ceteris paribus, the smaller the var $(R_{it}|\psi_{t-1})$ is, the larger R_i^2 will be, and this variance is smaller for the bond data. The most dramatic differences between the results for the stock return models 1 through 6 in table 1 and the Treasury bill models are the χ^2 statistics. For the Treasury bill models, the marginal significance levels are essentially zero, providing strong evidence against the restrictions.

Grossman and Shiller (1980) have examined the implications of the same multiperiod models of asset returns that are considered here. They noted that the parameters α and β can be identified and estimated from unconditional means and covariances of the logarithms of returns on two assets and aggregate consumption. Using yearly observations, they found values of $|\hat{\alpha}|$ substantially greater than one with correspondingly large standard errors for a variety of sample periods, including samples confined to the postwar period. For the

TABLE 4

SUMMARY OF MAXIMUM LIKELIHOOD RESULTS FOR NOMINAL RISK-FREE RETURN

Model	$\hat{\alpha}$*	$\hat{\beta}$*	CONS	NLAG	R_c^2	R_R^2	χ^2†	df
1	−.164	.9997	ND	2	.218	.130	27.34	3
	(.056)	(.0002)					(.9999)	
2	−.188	.9998	ND	4	.212	.152	33.48	7
	(.060)	(.0002)					(.9999)	
3	−.931	1.0015	NDS	2	.128	.181	30.08	3
	(.044)	(.0004)					(.9999)	
4	−1.289	1.0022	NDS	4	.131	.198	30.82	7
	(.088)	(.0006)					(.9999)	

* Standard errors are indicated in parentheses.
† Probability values are indicated in parentheses.

TABLE 5

Summary of Maximum Likelihood Results for Nominal Risk-Free and Value-Weighted Returns

Model	$\hat{\alpha}$*	$\hat{\beta}$*	CONS	NLAG	χ^2†	df
1	−30.58 (34.06)	1.001 (.0462)	ND	0	Just identified	Just identified
2	−.205	.999	ND	4	170.25 (.9999)	24
3	−58.25 (66.57)	1.088 (.0687)	NDS	0	Just identified	Just identified
4	−.209	1.000	NDS	4	366.22 (.9999)	24

* Standard errors are indicated in parentheses.
† Probability values are indicated in parentheses.

sake of comparison, we have computed maximum likelihood estimates of α and β using monthly observations on the value-weighted New York Stock Exchange return, the 1-month Treasury bill return, and the consumption of nondurables for values of NLAG equal to 0 and 4. These results are reported in table 5. The estimation procedure employed by Grossman and Shiller corresponds to the case when NLAG = 0. Consistent with their results, we found $|\hat{\alpha}|$ to be very large with a correspondingly large standard error when NLAG = 0. Consistent with our other findings, $|\hat{\alpha}|$ is approximately one when the serial correlation in the time-series data is taken into account in estimation. This shows the extent to which the precision and magnitude of our estimates rely on the restrictions across the serial correlation parameters of the respective time series.

By simultaneously considering more than one asset, we can test CRRA-lognormal models using nominal returns without having to measure aggregate consumption and the implicit consumption deflator. These tests remain valid in the presence of multiplicative shocks to preferences. To see this, consider the following generalization of the CRRA period utility function,

$$U(c_t, \lambda_t) = \frac{c_t^{\gamma}\lambda_t}{\gamma},$$

where λ_t is a (possibly degenerate) random shock observed by agents at time t that can be serially correlated. For this set of preferences, condition (5) becomes

$$E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{\alpha}\left(\frac{\lambda_{t+1}}{\lambda_t}\right)r_{it+1}\right] = 1; \quad i = 1, \ldots, N. \tag{19}$$

Under the assumption that $\{(\log \lambda_t - \log \lambda_{t-1}, \mathbf{Y}_t')'\}$ is a stationary Gaussian process, relation (9) becomes

$$\tilde{V}_{it} = \alpha X_{it} + R_{it} + \log \lambda_t - \log \lambda_{t-1} + \log \beta + (\tilde{\sigma}_i^2/2),\ i = 1, \ldots, n,$$

where $E(\tilde{V}_{it}|\tilde{\psi}_{t-1}) = 0$, $\tilde{\psi}_{t-1} = \psi_{t-1} U\{\lambda_{t-s}: s \geq 1\}$, and $\tilde{\sigma}_i^2$ is the conditional variance of $\tilde{V}_{it}$. Thus, the difference between the logarithms of any two real returns is

$$R_{lt} - R_{jt} = (\tilde{\sigma}_j^2/2) - (\tilde{\sigma}_l^2/2) + V_{lt} - V_{jt}. \tag{20}$$

Since the difference $R_{lt} - R_{jt}$ equals the difference between the logarithms of the nominal returns (say, $\tilde{R}_{lt} - \tilde{R}_{jt}$) and the error $V_{lt} - V_{jt}$ is orthogonal to the elements of the information set $\tilde{\psi}_{t-1}$, (20) implies that $\tilde{R}_{lt} - \tilde{R}_{jt}$ must be uncorrelated with the elements of $\tilde{\psi}_{t-1}$ if the model is true. Therefore, the model can be tested by determining whether the slope coefficients in regressions of the difference between the logarithms of nominal returns onto variables in $\tilde{\psi}_{t-1}$ are significantly different from zero.

These tests also avoid some of the timing problems in aligning the consumption data with the return data alluded to earlier. For instance, relation (19) is also implied by a continuous time asset pricing model in which c_t is the instantaneous real per capita consumption flow, r_{it} is the return on asset i over the time interval $(t - 1, t]$, and β is related to the continuous time rate of time preference $\dot{\rho}$ via $e^{-\rho} = \beta$. Since the tests do not require observations on consumption, we are free to interpret them as tests of either discrete or continuous-time specifications. A drawback of not using consumption data is that the preference parameters α and β can no longer be identified.

We conducted these tests as follows. Let $\tilde{R}_{1t}$ denote the logarithm of the nominal Treasury bill return, $\Delta\tilde{R}_{1t} = \tilde{R}_{1t} - \tilde{R}_{1t-1}$, and $\mathbf{R}_t = (\tilde{R}_{2t} - \tilde{R}_{1t}, \tilde{R}_{3t} - \tilde{R}_{1t}, \ldots, \tilde{R}_{nt} - \tilde{R}_{1t})'$. The system of regression equations $\tilde{\mathbf{R}}_t = \tilde{\boldsymbol{\mu}} + \tilde{A}_1(L)\Delta\tilde{R}_{1t} + \tilde{A}_2(L)\tilde{\mathbf{R}}_{t-1} + \tilde{\mathbf{U}}_t$ was estimated using equation-by-equation ordinary least squares, where $\tilde{\boldsymbol{\mu}}$ is an $n - 1$ dimensional vector of constants, $\tilde{A}_1(L)$ is an $n - 1$ dimensional vector lag polynomial of order NLAG, and $\tilde{A}_2(L)$ is an $n - 1$ dimensional matrix lag polynomial of order NLAG $- 1$. Since the nominal Treasury bill return is risk free, $\Delta\tilde{R}_{1t}$ is known to agents at date $t - 1$, and it was therefore included as a right-hand-side variable in the regression equations. Likelihood ratio statistics were calculated to test the restriction $\tilde{A}_1(L) = 0$ and $\tilde{A}_2(L) = 0$ implied by CRRA-lognormal models. Two choices of returns corresponding to two different models were used in conducting the tests. For the first model, we used two returns, the second being the nominal value-weighted stock return. For the second model, we used four returns, the last three being the nominal stock returns on three Dow Jones Industrial stocks: American Brand, Exxon, and IBM. In both cases, NLAG was set equal to 2. The likelihood ratio statistics for the aggregate and individual return models are $\chi^2(5) = 16.56$ and $\chi^2(27) = 53.19$, respectively. The associated probability values are .9946 and .9981, and thus the restrictions are rejected by the data except at extremely low significance levels.

IV. Concluding Remarks

In this paper we have derived a time-series representation of consumption and asset returns that characterizes the restrictions on the temporal covariance structure of these variables implied by a class of general-equilibrium asset pricing models with time-separable, constant relative risk-averse preferences in which consumption and returns are lognormally distributed. Maximum likelihood estimation of the free parameters of most of the monthly models yielded point estimates of the coefficient of relative risk aversion that were between zero and two. The test statistics provided little evidence against the models using the value-weighted return on stocks listed on the New York exchange. In contrast, the marginal significance levels of the test statistics for the models of individual Dow Jones and Treasury bill returns were essentially zero. We also conducted tests of CRRA-lognormal models using multiple returns that are robust to mismeasurement of consumption and the deflator and accommodate certain types of shocks to preferences. These tests provided substantial evidence against the restrictions as well. In light of results reported here, we plan on pursuing models of asset returns with more general specifications of preferences and distribution-free methods of estimation and inference (Eichenbaum, Hansen, and Singleton 1982; Hansen and Singleton 1982).

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