

**CONSUMPTION, ASSET MARKETS, AND
MACROECONOMIC FLUCTUATIONS**
A Comment*

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I. INTRODUCTION

This paper by Shiller and recent research by Grossman and Shiller (1980, 1981) serve as a useful starting point for examining empirically the relationship between intertemporal marginal rates of substitution and returns on assets. Although a rich body of theoretical work studying intertemporal capital asset pricing models emphasizes this relationship, literature which examines the empirical implications of these models is sparse. Grossman and Shiller (1981) have provided some interesting results linking the behavior of aggregate consumption and asset returns, and Shiller has offered new insights in this current paper.

My comment on Shiller's paper will be organized as follows. Section II provides brief remarks about the existing evidence on the volatility of asset prices. Section III describes the impact of introducing uncertainty into empirical analyses. Section II extends Shiller's discussion on bounds on the variance of the intertemporal marginal rate of substitution. Finally, section V discusses some specification issues involved in undertaking formal econometric estimation and inference in analyzing uncertain versions of the dynamic model studied by Shiller.

II. EXISTING EVIDENCE ON THE PRESENT VALUE RELATION

In Shiller's work on volatility, there has been little emphasis on formal statistical inference. The analysis in this paper is described by the author as exploratory. While acknowledging that such an analysis is useful, I would like

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to emphasize the potential danger of focusing on population characteristics of the data that can be inferred only very imprecisely

As an illustration of why this concern might be important, consider the work of Leroy and Porter (1981) that studied the present value relation under risk neutrality. The reason for this consideration is not because of any *a priori* appeal of constant marginal rates of substitution, but rather because Leroy and Porter undertook formal statistical inference in studying implied variance bounds. Among other things they compare the perfect foresight stock-price variance, using Standard & Poors stock-price data in the post-war period. Although their data set differs from Shiller's, they did find the perfect foresight price-variance estimate to be nineteen times less than the actual price-variance estimate. However, this apparently dramatic evidence against the model under risk neutrality becomes innocuous when the standardized normal test statistic is calculated to be only -.24. That is, there is very little information in the sample about this particular variance comparison. To be fair, Leroy and Porter do find substantial evidence against the risk neutral specification in other tests they perform.

A related concern is the possibility of nontrivial stochastic growth components in the price and dividend series. Short of modeling growth in a serious way, one has to be cautious in overinterpreting observed dividend and price behavior using covariance stationary models. Shiller notes that the implied variances under risk neutrality are violated when stationarity is imposed only on the first differences of prices and dividends. It turns out, however, that these first differences inequalities are violated only when the differencing time interval is expanded sufficiently far. Unfortunately, as one expands the differencing time interval, accurate estimation of the variances becomes more difficult.

III. THE ROLE OF UNCERTAINTY

One of the modes of analysis used by Shiller is to compare the observed time-paths of certain variables with the time-paths implied by a perfect foresight version of a dynamic model. In the dynamic model, a representative consumer chooses sequences of a consumption good and asset holdings by maximizing a time-separable utility-functional subject to a budget constraint. While the perfect foresight specification is an interesting limiting case, it is also important that we understand the impact of introducing uncertainty into the analysis.

Shiller reports results to indicate that when the hypothetical consumer's preference parameter, A , is four, the implied perfect foresight time-path

for the Standard & Poors Stock Price Index is closer to the observed time-path than when A is zero. This is especially true in the prewar period. If the perfect foresight version of the model is taken literally, then the two time-paths should coincide. In the absence of such an exact fit, we need to ask ourselves what possible reasons explain these deviations. While there are several alternative explanations—for instance, measurement errors or misspecification of preferences—the presence of uncertainty in the underlying economic environment is a natural candidate.

In an uncertain environment, the deviations between the perfect foresight stock-price path and the actual path should be autocorrelated. Consequently, persistent differences between the two time-paths are consistent with the theoretical economic model. Calibrating the model by matching up these price-paths becomes difficult, and formal procedures such as minimizing sums of squared deviations are hard to justify. However, superior procedures that allow for uncertainty exist for estimating the preference parameters. For references see Grossman and Shiller (1980), Hall (1981), and Hansen and Singleton (1981, 1982).

In his perfect foresight analysis, Shiller obtains a linearized version of the model that takes the form

$$P_t = K \sum_{k=1}^{\infty} \delta^k \Delta C_{t+k} + \sum_{k=1}^{\infty} \delta^k D_{t+k} \quad (1)$$

where P_t is the stock price at time t , C_t is per capita consumption at time t , D_t is the dividend payment at time t , δ is a discount factor satisfying $0 < \delta < 1$, and K is a constant that depends on preference parameters and on the population means of consumption and dividends.¹ Shiller then studies the spectral characteristics of the linear filter

$$a(L^{-1}) = K \sum_{k=1}^{\infty} \delta^k L^{-k} (1-L) = \frac{K(1-L)\delta L^{-1}}{(1-\delta L^{-1})}$$

that links consumption and stock prices in (1). Here L denotes the lag operator. Our next task is to consider the impact of uncertainty on this link between consumption and stock prices.

If we ignore the possibly substantial approximation error associated

¹See Shiller's equation (8). Throughout this analysis it is presumed that detrended versions of P_t , C_t , and D_t can be appropriately modeled as covariance stationary processes. The presence of non-trivial, possibly stochastic, growth components will contaminate the analysis.

with the linearization and if we introduce uncertainty into the analysis, then the linearized model becomes

$$P_t = E[a(L^{-1})C_t|\Omega_t] + E\left(\frac{\delta L^{-1}}{1-\delta L^{-1}} D_t|\Omega_t\right)$$

where Ω_t denotes the information set of the representative agent. For simplicity we shall abstain from considering dividend uncertainty, and we shall adopt a relatively simple specification of consumption uncertainty. More precisely, suppose that consumption has a moving average representation:

$$c_t = \beta(L)u_t$$

where $\{u_t: -\infty < t < +\infty\}$ is a scalar white noise with a unit variance, $\beta(L) = \sum_{j=0}^{\infty} \beta_j Z^j$ with $\sum_{j=0}^{\infty} \beta_j^2 < +\infty$ and $E(u_{t+1}|\Omega_t) = 0$.² Here c_t denotes the deviation of consumption from its population mean.

With this specification of uncertainty, it is straightforward to calculate $E[a(L^{-1})c_t|\Omega_t]$. Using methods described in Hansen and Sargent (1980), it can be shown that

$$p_t = E[a(L^{-1})c_t|\Omega_t] = \gamma(L)u_t$$

where

$$\gamma(Z) = \frac{K(1-Z)\delta\beta(Z) - K(1-\delta)\delta\beta(\delta)}{(Z-\delta)} \quad (2)$$

When $\beta(L)$ has an inverse that is possibly two-sided, then the linear filter linking p_t and c_t is given by $p_t = \theta(L)c_t$ where

$$\theta(Z) = \frac{\gamma(Z)}{\beta(Z)} = a(Z^{-1}) - \frac{K(1-\delta)\delta\beta(\delta)}{(Z-\delta)\beta(Z)} \quad (3)$$

²Note that we allow $\beta(Z)$ to have zeroes inside the unit circle.

³A necessary and sufficient condition for $\beta(Z)$ to have an inverse when $\beta(Z)$ is a rational function of Z is that the zeroes of $\beta(Z)$ do lie on the unit circle.

The transfer function $\theta(Z)$ is equal to $a(Z^{-1})$ only when $\beta(\delta)$ is zero, in which case it can be shown that the random variable

$$c_t^* = a(L^{-1})c_t$$

is contained in the information set Ω_t .

When $\beta(\delta)$ is different from zero, the spectral characteristics of the transforms $\theta(Z)$ and $a(Z)$ may differ substantially. In the absence of restrictions on $\beta(Z)$, relation (3) places only mild restrictions on the function $\theta(Z)$. To understand this point, we consider a rational specification of $\theta(Z)$ given by

$$\theta(Z) = \frac{\theta_1(Z)}{\theta_2(Z)}$$

where $\theta_1(Z)$ and $\theta_2(Z)$ are finite order polynomials that do not have zeroes in common and where $\theta_2(Z)$ does not have any zeroes on the unit circle or at δ . Solving (3) for the implied function $\beta(Z)$, we obtain

$$\beta(Z) = \frac{\beta_1(Z)}{\beta_2(Z)}$$

where

$$\beta_1(Z) = \theta_2(Z)$$

$$\beta_2(Z) = [K(1-Z)\delta\theta_2(Z) - \theta_1(Z)(Z-\delta)]C$$

where C is an arbitrary constant. For the operator $\beta(L)$ to be one-sided, it must be that if $\beta_2(Z^*) = 0$ for $|Z^*| \leq 1$, then $\beta_1(Z^*) = 0$. This means the $\theta_1(Z)$ and $\theta_2(Z)$ must satisfy the requirement that

$$K(1-Z)\delta\theta_2(Z) \neq \theta_1(Z)(Z-\delta) \tag{4}$$

for all $|Z| \leq 1$.

There is a rich class of $\theta_1(Z)$'s and $\theta_2(Z)$'s which will satisfy (4). For any choice of $\theta_2(Z)$ that does not have zeroes on the unit circle or at δ , one can find admissible choices of $\theta_1(Z)$. Similarly, for any choice of $\theta_1(Z)$ one can find admissible choices of $\theta_2(Z)$. Consequently, the location of poles

or zeroes of $\theta(Z)$ can be virtually arbitrary. In uncertain environments the linear filter linking p_t and c_t cannot be characterized independently of the stochastic law of motion for consumption. Consistent with the message in the linear, rational expectations literature, the more appropriate mode of analysis is to study the implied cross-equation restrictions, in this case the restrictions across the moving average representations for p_t and c_t . In summary, spectral analysis of linear filters linking consumption and stock prices has little merit in uncertain environments without a simultaneous analysis of the stochastic law of motion for consumption.

IV. BOUNDS ON THE VARIANCES OF THE INTERTEMPORAL MARGINAL RATE OF SUBSTITUTION

Shiller deduces a lower bound on the variance of the intertemporal marginal rate of substitution (IMRS) that is appropriate for uncertain versions of the dynamic representative consumer model. His lower bound has the virtue that it can be represented in terms of the means and variances of two returns, but he notes that the bounds can be made sharper by exploiting the covariation of the two returns. In this section, I show how to obtain a lower bound on the IMRS that not only exploits this covariation but also allows one to incorporate more than two returns into the analysis. Computation of this lower bound relies on a formula that Roll (1977) used to deduce the mean-variance frontier for a vector of primitive returns.

Suppose the econometrician has observations on a vector of returns $r = (r_1, r_2, \dots, r_n)'$ on securities where time subscripts are suppressed for notational convenience. Let s denote the IMRS for the representative consumer, and let J be a subset of the information of this consumer on the date on which the securities are traded. This subset is to be specified by the econometrician *a priori* and can consist of only constant random variables. An implication of the model is that

$$E(r_j s | J) = 1 \quad \text{for } j = 1, \dots, n. \quad (5)$$

(see Shiller's equation (3).) Suppose that at least two elements of $E(r|J)$ are almost surely different and that the conditional covariance matrix, $\text{Var}(r|J)$, is almost surely nonsingular. We presume that the econometrician can deduce $E(r|J)$ and $\text{Var}(r|J)$. The most tractable specification of J is the set of constant random variables, in which case unconditional means and covariances of returns

are all that are required. Under the preceding set of assumptions, there exists an n dimensional random vector W contained in J (a vector of constants when J is the set of constant random variables) such that

$$E(W'r|J) = W'E(r|J) = 0 \quad (6)$$

and $W'1_n = 1$

where 1_n is n dimensional vector of ones. Typically, there will be infinitely many such W 's. The random variable $W'r = r_w$ is a return on a portfolio constructed from elements of r . Relation (5) implies that this return satisfies

$$E(r_w s|J) = 1 \quad (7)$$

Taking a covariance decomposition of (7) and using the fact that $E(r_w|J) = 0$, it follows that

$$\text{Cov}(r_w, s|J) = 1 \quad (7)$$

This relation in turn implies the inequality

$$\text{Var}(s|J) \geq \frac{1}{\text{Var}(r_w|J)} \quad (8)$$

To make inequality (8) as sharp as possible we find the portfolio of returns in r that has the smallest conditional variance among the class of portfolios with expected returns equal to zero. That is, we find the return on the conditional mean-variance frontier that has a conditional mean equal to zero. From Corollary 1 of Roll (1977), this return, r_w^* has conditional variance

$$\text{Var}(r_w^*|J) = a/(ac-b^2) \quad (9)$$

where

$$a = E(r|J)' \text{Var}(r|J)^{-1} E(r|J)$$

$$b = E(r|J)' \text{Var}(r|J)^{-1} 1_n$$

$$c = 1_n' \text{Var}(r|J)^{-1} 1_n$$

Consequently, we obtain the inequality

$$\text{Var}(s|J) \geq \frac{1}{\text{Var}(r_w^*|J)}.$$

When there are only two returns in r , inequality (9) is sharper than the one proposed by Shiller in that it exploits the covariation of the two returns as Shiller indicates possible. To see this, observe that when n is two there will be only one vector W that satisfies (6). This vector is given by

$$W^* = \frac{1}{E(r_1|J) - E(r_2|J)} \begin{bmatrix} -E(r_2|J) \\ E(r_1|J) \end{bmatrix}$$

The conditional variance of $r_w^* = W^* r$ satisfies the inequality

$$\text{Var}(r_w^*|J)^{1/2} \leq \frac{|E(r_2|J)|\text{Var}(r_1|J)^{1/2} + |E(r_1|J)|\text{Var}(r_2|J)^{1/2}}{|E(r_1|J) - E(r_2|J)|} \quad (10)$$

Combining relations (9) and (10) yields

$$\text{Var}(s|J)^{1/2} \geq \frac{1}{\text{Var}(r_w^*|J)^{1/2}} \geq \frac{|E(r_1|J) - E(r_2|J)|}{|E(r_2|J)|\text{Var}(r_1|J)^{1/2} + |E(r_1|J)|\text{Var}(r_2|J)^{1/2}}$$

By ignoring the middle term, one obtains the lower bound on $\text{Var}(s|J)^{1/2}$ suggested by Shiller. Inequality (9) becomes even sharper when there are more than two returns in r .

The use of variance bounds for devising statistical tests of the representative consumer model requires some means of estimating the variance of the IMRS. When it is possible to obtain measurements and/or observations on the IMRS, more direct tests of the model can be devised that simultaneously test restrictions other than the variance inequalities described above. In such circumstances, a natural question is why consider variance bounds testing. Two responses to this query have been suggested:

- (1) variance bounds tests are computationally more tractable;
- (2) variance bounds tests do not require that the data be correctly aligned.

In considering the first response, it is important to recognize that while calcu-

lating point estimates of variances is easy, engaging in formal statistical inference is more involved. Even appealing to asymptotic distribution theory, one has to estimate consistently the asymptotic covariance matrix of the vector of variance estimators. In the dynamic model studied by Shiller, there is no *a priori* reason to rule out serial correlation in the returns and the IMRS. A joint normality assumption of the underlying variables is undesirable given that IMRS is greater than or equal to zero.⁴ While these difficulties are, in principle, surmountable, one should not overstate the simplicity of the variance bounds tests. In considering the second response, I wonder whether it really is possible to obtain accurate information about the IMRS without knowing how to correctly align these data with return data. Most arguments against being able to correctly align the data also undermine the accuracy of the measurements of the IMRS.

V. SPECIFICATION ISSUES IN ECONOMETRIC ANALYSES

As noted by Shiller, one of the biggest obstacles in conducting econometric analyses of dynamic representative consumer models is obtaining reliable measurements of the IMRS. Even though Hansen and Singleton (1982) have proposed some distribution-based tests that avoid such measurements, it is still desirable for formal econometric analyses to link observed consumption behavior with asset price behavior. This means that a researcher must confront a variety of possible specification problems. Shiller mentions the possibility of errors in the measurement of aggregate consumption and the difficulty associated with aggregating over heterogeneous agents. In addition to these problems, a researcher must face three other important specification issues.

First, one must specify the appropriate decision time interval or period for the economic agents. For instance, if a discrete time model is being considered, the question arises about how frequently consumption decisions are made relative to the time intervals between observations available to an econometrician. The most convenient assumption and the one adopted for much of Shiller's paper and used by Grossman and Shiller (1981) and Hansen and Singleton (1981, 1982) is that the time interval between consumption decisions is the same as the time interval between consumption observations employed in the empirical analyses. For Shiller and Grossman and Shiller (1981), this time interval is one year, and for Hansen and Singleton (1981, 1982) it is one month. While this assumption is the most convenient for econometric analysis, it is not necessarily the most appealing from a theoretical standpoint. An al-

⁴See Singleton (1980) and Leroy and Porter (1981) for discussion of how to do variance bounds tests in other contexts. Their analyses assume that fourth-order cumulants are zero as is the case when the random variables are jointly normal, although this assumption can be relaxed.

ternative strategy is to follow the lead of Marsh (1980) by analyzing continuous time versions of the representative consumer model, treating the consumption observations as integrals or averages over time of instantaneous consumption decisions.

A second specification issue is to determine the appropriate decision vector of the representative agent. This involves judgments about the level of aggregation across commodities appropriate for analyzing macro time series. It is desirable to include the service flow from the stock of durable goods in the analysis. The dynamic models of Lucas and Rapping (1969) and Kydland and Prescott (1981) emphasize the importance of the labor-leisure allocation of consumers in explaining macroeconomic fluctuations. Consequently, it would be of interest to incorporate leisure into the measures of the IMRS.

A third specification issue is the parameterization of the preferences of a representative consumer. While one can blend estimation of preference parameters with statistical inference, the obvious tension emerges between the size of the class of admissible preferences and the number of overidentifying restrictions that can be tested. In the absence of *a priori* restrictions on preferences beyond concavity, it is not obvious that this intertemporal representative consumer model has much empirical content. Perhaps the task posed to quantitative macroeconomists should be to ascertain empirically plausible preference specifications rather than to test the intertemporal theory. Even if one is willing to specify restrictive preference families *a priori*, measuring the IMRS is particularly difficult if these preferences are allowed to be nonseparable over time. One needs information not only about marginal utilities but also about the conditional expectations of the marginal utilities. However, the limited information, instrumental variables procedures proposed by Hansen and Singleton (1981) can be used to estimate preference parameters without determining the conditional expectations of these marginal utilities. See Hansen, Richard, and Singleton (1982) for a discussion.

All of these points merely suggest that there remain challenging tasks facing quantitative macroeconomists interested in studying dynamic representative consumer models.

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