Valuation Dynamics
in Models with Financial Frictions

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Research Objective

Compare/contrast implications of DSGE models with financial frictions through the study of their nonlinear transition mechanisms

▷ Environment
  Continuous time with Brownian shocks
  Financial intermediaries
  Heterogeneous productivity or market access

▷ Comparison Targets
  Macroeconomic quantity implications
  Asset pricing implications
  Macro- and micro-prudential policies
Challenges

Model features:

▷ Nonlinear transition mechanism
▷ Some shock configurations have big consequences
▷ Endogenous transitions across two regimes: (one where sector wide constraints are binding and another when they are not)

Model assessments:

▷ Alter or extend linear methods of analysis
▷ Perform cross model comparisons
Approaches

▷ Opening the black box: structural approach hold fixed some aspects of the economic environment (including parameters) while changing others and exploring implications

▷ Imposing observational constraints: holding fixed some implications and changing parameters accordingly to match these while altering the economic environment
“Nesting” Model

▷ Technology
  ○ A-K production function with $a_e \geq a_h$ and adjustment costs
  ○ (total factor) productivity shocks
  ○ growth rate and stochastic vol shocks (long-run risk)
  ○ idiosyncratic shocks

▷ Markets
  ○ capital traded with shorting constraint at a price $Q_t$
  ○ experts face a skin-in-the-game constraint where the fraction of held capital, $\chi_t$, is restricted ($\chi_t \geq \chi$)

▷ Preferences
  ○ recursive utility, discount rate $\delta$, IES $\frac{1}{\rho}$, and risk aversion $\gamma$
  ○ different preferences with $\gamma_h \geq \gamma_e$
  ○ OLG for technical reasons
Models Nested

▷ Complete markets with long run risk
  Bansal & Yaron (2004)
  Hansen, Heaton & Li (2008)
  Eberly & Wang (2011)

▷ Complete markets with heterogeneous preferences
  Longstaff & Wang (2012)
  Garleanu & Panageas (2015)

▷ Incomplete market/limited participation
  Basak & Cuoco (1998)
  Kogan & Makarov & Uppal (2007)
  He & Krishnamurthy (2012)

▷ Incomplete market/capital misallocation
  Brunnermeier & Sannikov (2014)

▷ Complete markets for aggregate risk and stochastic volatility
  Di Tella (2017)
Diagnostic Tools I

▶ Quantities
  ○ consumption/wealth ratio
  ○ investment rate
  ○ output growth

▶ Prices
  ○ risk-free rate
  ○ risk-price vectors (one per agent)
  ○ capital price

▶ State dynamics
  ○ drift and diffusion of the aggregate state vector
  ○ ergodic density of state vector
Models of Asset Valuation

Two channels:

- **Stochastic growth** modeled as a process \( G = \{ G_t \} \) where \( G_t \) captures growth between dates zero and \( t \).
- **Stochastic discounting** modeled as a process \( S = \{ S_t \} \) where \( S_t \) assigns risk-adjusted prices to cash flows at date \( t \).

Date zero prices of a payoff \( G_t \) are

\[
\pi = \mathbb{E} (S_t G_t | \mathcal{F}_0)
\]

where \( X_0 \) captures current period information.

**Stochastic discounting** reflects investor preferences through the intertemporal marginal rate of substitution for marginal investors.
Impulse Problem

Ragnar Frisch (1933):

There are several alternative ways in which one may approach the impulse problem .... One way which I believe is particularly fruitful and promising is to study what would become of the solution of a determinate dynamic system if it were exposed to a stream of erratic shocks that constantly upsets the continuous evolution, and by so doing introduces into the system the energy necessary to maintain the swings.

Irving Fisher (1930):

The manner in which risk operates upon time preference will differ, among other things, according to the particular periods in the future to which the risk applies.
Diagnostic Tools II

Transition dynamics and valuation through altering cash flow exposure to shocks.

▷ Study implication on the price today of changing the exposure tomorrow on a cash flow at some future date.
▷ Represent shock price elasticities by normalizing the exposure and studying the impact on the logarithms of the expected returns.
▷ Construct pricing counterpart to impulse response functions.
Unpack the Term Structure of Risk Premia!

Counterparts to impulse response functions pertinent to valuation:

- shock-exposure elasticities
- shock-price elasticities

These are the ingredients to risk premia, and they have a term structure induced by the changes in the investment horizons.

Construct Elasticities

▷ Construct shock elasticities as counterparts to impulse response functions

▷ Use (exponential) martingale perturbation $D_{(\tau, \tau+s)}$ to an underlying positive (multiplicative) process $M$ where:

$$d \ln M_t = \mu_m(X_t) dt + \sigma_m(X_t) \cdot dW_t$$

$$\epsilon_m(x, t, \tau) : = \lim_{s \downarrow 0} \frac{d}{ds} \log \mathbb{E} \left[ \frac{M_t}{M_0} D_{(\tau, \tau+s)} | X_0 = x \right]$$

where $dW_t$ is a vector of Brownian increments.

▷ Apply to a cash-flow $G_t$ and stochastic discount factor $S_t$

- shock exposure elasticity $\epsilon_g(x, t, \tau)$;
- shock cost elasticity $\epsilon_{sg}(x, t, \tau)$;
- shock price elasticity $\epsilon_g(x, t, \tau) - \epsilon_{sg}(x, t, \tau)$

In what follows $\tau = 0$ or $\tau = t$. 
Interpret Elasticities

Recall

\[
\epsilon_m(x, t, \tau) = \lim_{s \downarrow 0} \frac{d}{ds} \log \mathbb{E} \left[ \frac{M_t}{M_0} D_{(\tau, \tau+s)} | X_0 = x \right]
\]

where \( D_{(\tau, \tau+s)} \) is an exponential martingale perturbation.

Two interpretations:

▷ Change in probability measure - local impulse response
▷ Change in cash flow exposure - local risk risk return

Depend on current state, horizon, and date when the perturbation occurs.
What do These Elasticities Contribute?

▷ What shocks investors do care about as measured by expected return compensation?
▷ How do these compensations vary across states and over horizons?
▷ How do the shadow compensation differ across agent type?
Overview of Solution Method

▷ **Markov equilibrium** – aggregate state vector $X_t$:
  - **exogenous states**: $Z_t$ (growth), $V_t$ (agg. stochastic vol.), and $\varsigma_t$ (idio. stochastic vol.)
  - **endogenous state**: $W_t := \frac{N_{e,t}}{N_{e,t} + N_{h,t}}$ (wealth share)

▷ “Value function” approach: $U_i (N_{i,t}, X_t) = N_{i,t}^{1-\gamma_i} \xi_i (X_t)$ and $N_{i,t}$ is the individual net wealth

▷ $(\xi_e, \xi_h)$ solutions to second order non-linear PDEs – implicit FD scheme with artificial time derivative

▷ Each time-step: compute aggregate state dynamics and prices using the value functions from the previous time-step

▷ Endogenous state partition due to occasionally-binding constraints

▷ Implementation in C++ allowing for HPC
Computation: Value Functions

Scaled value functions $\xi_i$ solve PDEs like

$$0 = K_i + A_i \xi_i + B_i \cdot \partial_x \xi_i + \text{trace}[C_i C_i' \partial_{xx'} \xi_i], \quad x = (w, z, v, \varsigma),$$

where the coefficients are:

$$K_i = K_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h)$$
$$A_i = A_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h)$$
$$B_i = B_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h)$$
$$C_i = C_i(x, \xi_e, \xi_h, \partial_x \xi_e, \partial_x \xi_h)$$

The dependence of $A, B, C$ on $(\xi_e, \xi_h)$ arises due to general equilibrium.

We solve this PDE system with an iterative approach with two steps:

- given coefficients, we solve the linear PDE and obtain $\{\xi_i\}_{i=e,h}$
- given PDE solution $\{\xi_i\}_{i=e,h}$, we update coefficients using equilibrium constraints
Computation: Constraints

Capital distribution $\kappa \in [0, 1]$ and expert equity issuance $\chi \in [\underline{\chi}, 1]$ determine the occasionally-binding constraints of the models:

$$0 = \min(1 - \kappa, -\alpha_h)$$

$$0 = \min(\chi - \underline{\chi}, \alpha_e),$$

where $\alpha_i$ is agent $i$’s endogenous premium on capital.

**Economic intuition.**

▷ Experts hold all capital ($\kappa = 1$) if and only if households obtain no premium for holding it ($\alpha_h < 0$)

▷ Experts issue as much equity as possible ($\chi = \underline{\chi}$) if and only if their inside equity compensation exceeds the outside equity compensation ($\alpha_e > 0$)
Computation: Constraints

Variational inequalities. Algebraic equations on part of the state space (when constraints bind) and first-order non-linear elliptic PDEs on the complement (when constraints are slack).

\[ 0 = \min(1 - \kappa, -\alpha_h) \]
\[ 0 = \min(\chi - \underline{\chi}, \alpha_e), \]

where

\[ \alpha_h = F_h(x, \kappa, \partial_x \kappa, \chi, \partial_x \chi) \]
\[ \alpha_e = F_e(x, \kappa, \partial_x \kappa, \chi, \partial_x \chi). \]

Solution method. Use “explicit” first difference scheme with false transient. See Oberman (2006)

\[ \frac{\kappa^{t+\Delta} - \kappa^t}{\Delta} = \min \left[ 1 - \kappa^t, F_h(x, \kappa^t, \partial_x \kappa^t, \chi^t, \partial_x \chi^t) \right] \]
Binding Constraints

- When does the constraint always bind?

- Economic setting
  - experts are the only producers
  - skin-in-the-game constraint $\chi \geq \chi_0 = .5$
  - TFP shocks only
  - EIS = 1

- Compare homogeneous RRA ($\gamma_e = \gamma_h$) to heterogeneous RRA ($\gamma_e < \gamma_h$)
Binding Constraints

Expert’s skin-in-the-game $\chi$ in the two models.
Binding Constraints

Expert’s shadow risk prices $\pi_e$ in the two models.
Shocks and Financial Frictions

▷ How do financial frictions affect agents’ attitudes about shocks?

▷ Economic setting of focus
  ○ experts are the only producers
  ○ shocks to productivity, growth rate, and volatility
  ○ RRA = 3, EIS = 1

▷ Compare model with a skin in the game constraint ($\chi \geq \underline{\chi} = .5$) vs. model without frictions ($\chi = 0$)
Shocks and Financial Frictions

\[ \chi = .5 \]

\[ \chi = 0 \]

Expert’s and household’s productivity risk prices \( \pi_e, \pi_h \).
Heterogeneity: Productivity vs. Risk Aversion

▷ “Expert” agents in the economy are either more productive or less risk averse?

▷ Economic setting of focus
  ○ experts and households can both produce
  ○ no equity-issuance $\chi \equiv \underline{\chi} = 1$
  ○ shocks to TFP level, growth rate, and volatility
  ○ EIS = 1

▷ Compare an economy with differences in productivity ($a_e > a_h$ but $\gamma_e = \gamma_h = 3$) to one in with differences in risk aversion ($\gamma_e = 2, \gamma_h = 8$ but $a_e = a_h$)
Heterogeneity: Productivity vs. Risk Aversion

\[ a_e > a_h, \gamma_e = \gamma_h \quad \quad \quad \quad \quad \gamma_e < \gamma_h, a_e = a_h \]

Relative capital distribution \( \kappa \).
Heterogeneity: Productivity vs. Risk Aversion

\[ a_e > a_h, \gamma_e = \gamma_h \]

\[ \gamma_e < \gamma_h, a_e = a_h \]

Expert’s TFP risk price \( \pi_e \) in the two models.
Heterogeneity: Productivity vs. Risk Aversion

\[ a_e > a_h, \gamma_e = \gamma_h \]

Expert's productivity risk price \( \pi_e \) in the two models.
Heterogeneity: Productivity vs. Risk Aversion

\[ a_e > a_h, \gamma_e = \gamma_h \]

\[ \gamma_e < \gamma_h, a_e = a_h \]

Productivity shock-exposure elasticities
Heterogeneity: Productivity vs. Risk Aversion

\[ a_e > a_h, \gamma_e = \gamma_h \]

\[ \gamma_e < \gamma_h, a_e = a_h \]

Productivity shock price elasticities for experts.
Heterogeneity: Productivity vs. Risk Aversion

\[ a_e > a_h, \gamma_e = \gamma_h \]

\[ \gamma_e < \gamma_h, a_e = a_h \]

Productivity shock price elasticities for households.
Heterogeneity: Productivity vs. Risk Aversion

\[ a_e > a_h, \gamma_e = \gamma_h \]  

\[ \gamma_e < \gamma_h, a_e = a_h \]

Growth-rate shock exposure elasticities for aggregate consumption.
Heterogeneity: Productivity vs. Risk Aversion

\[ a_e > a_h, \gamma_e = \gamma_h \]

\[ \gamma_e < \gamma_h, a_e = a_h \]

Growth-rate shock price elasticities for experts.
Heterogeneity: Productivity vs. Risk Aversion

\[ a_e > a_h, \gamma_e = \gamma_h \]

\[ \gamma_e < \gamma_h, a_e = a_h \]

Household growth-rate shock price elasticities.
Conclusion / Next Steps

▷ Consider additional types of financial constraints

▷ Compare to smooth within regime models that target different time periods

▷ Analyze link between heterogenous preference models, heterogenous belief models, financial frictions’ models

▷ Provide user-friendly web application to compare and contrast models...
Web Application

Select your desired constellation of models...
Web Application

Tabs separating outcomes for prices, dynamics, etc...
Technology

Efficiency units of capital $K_t$ follow

$$dK_t = K_t \left[ (Z_t + \nu_t - \delta) \, dt + \sqrt{V_t} \sigma \cdot dW_t \right]$$

Exogenous state variables $(S_t, Z_t)$ follow

$$dZ_t = \lambda_z (\bar{z} - Z_t) \, dt + \sqrt{V_t} \sigma_v \cdot dW_t$$

$$dV_t = \lambda_v (\bar{v} - V_t) \, dt + \sqrt{V_t} \sigma_v \cdot dW_t$$

Adjustment costs: investment $\nu_t K_t \, dt$ costs $\Phi(\nu_t) k_t \, dt$ in output