Pricing Uncertainty Induced by Climate Change

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based on joint research with Michael Barnett and William Brock
previous paper available at SSRN: https://ssrn.com/abstract=3008833

Climate uncertainty

M. R. Allen *et al.*

“Global efforts to mitigate climate change are guided by projections of future temperatures. But the eventual equilibrium global mean temperature associated with a given stabilization level of atmospheric greenhouse gas concentrations remains uncertain, complicating the setting of stabilization targets to avoid potentially dangerous levels of global warming.” (2009)

American Physical Society Climate Change Statement Review

Modeling challenges

Quantitative policy assessment requires that we

- assess possibilities that are not well represented by historical evidence
- understand the interplay between climate and the economy
- measure the dynamic responses - emissions today impact the climate in the future
- confront uncertainty broadly conceived

Build and assess dynamic structural economic models

Tools

- decision theory under uncertainty
  - axiomatic defenses
  - recursive representations
- nonlinear impulse response functions
  changes in emissions today have impact on the climate in current and future time periods
- dynamic valuation and asset pricing
  confronting uncertainty in social or market valuation

Uncertainty components

A) risk -
uncertainty within a model: uncertain outcomes with known probabilities

B) ambiguity -
uncertainty across models: unknown weights for alternative possible models

C) misspecification -
uncertainty about models: unknown flaws of approximating models

Who is making the decisions?

A) econometricians -
construct estimators with good properties, produce posterior distributions, or bounds on posterior probabilities

B) economic agents -
forward-looking private agents confront uncertainty with implications for allocations and prices

C) policy makers -
design prudent policies in the face of uncertainty

While econometricians' primary goal may be to summarize evidence as it pertains to alternative models, economic agents and policy makers take actions whose consequences may differ depending on which model is the correct one.

Probability meets Social Science

Jacob Bernoulli (left)

Law of Large Numbers: how unknown probabilities are revealed (1713)

Dual Roles for Statistics in Economic Analysis

- ▷ Outside a model

  Given a dynamic economic model, researchers:

  - estimate unknown parameters
  - assess model implications

- ▷ Inside a model

  When *constructing* a dynamic economic model, researchers:

  - depict economic actors (consumers, enterprises) as they cope with uncertainty
  - deduce the consequences for market outcomes and resource allocations

Origins of decision theory

Blaise Pascal

Pascal's wager (published in Pensees in 1670): probabilities and consequences are intertwined in rational decision making

Decision theory

Alternative approaches

- dynamic variational preferences
- recursive multiple priors
- smooth ambiguity models

What do they provide

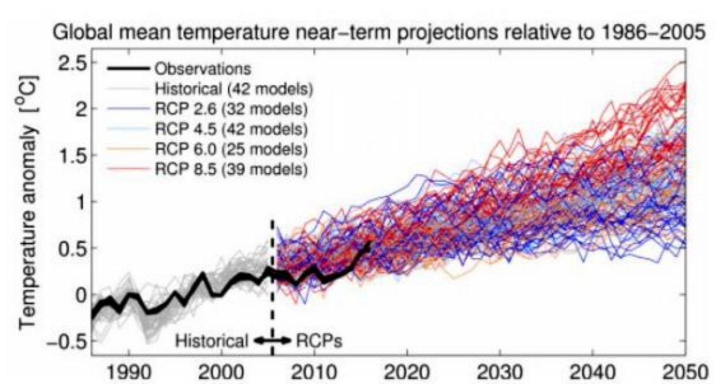
- tractable representations that capture alternative components to uncertainty.
- axiomatic defenses that add clarity to the applications

Misspecification and ambiguity aversion

In practice statistical models require subjective inputs and remain misspecified.

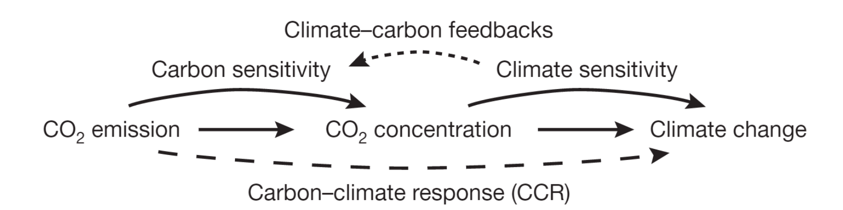
- Aim of robust approaches:
  - use models in sensible ways rather than discard them
  - use probability and statistics to provide tools for assessing sensitivity to subjective inputs and to potential misspecification
- Outcome of robust approaches:
  - target the ambiguity with the most adverse consequences for the decision maker.
  - induce caution by ambiguity about probability over future events and by potential model misspecification

Long-run model uncertainty of temperature changes



Source: Updated version of IPCC AR5 Figure 11.25a, showing observations and the CMIP5 model projections relative to 1986-2005. Provided at www.climate-lab-book.ac.uk.

Model simplification and potential misspecification



Source: Figure 1 from Matthews, Gillett and Zickfeld (2009).

CCR measures a long-term temperature response to emissions

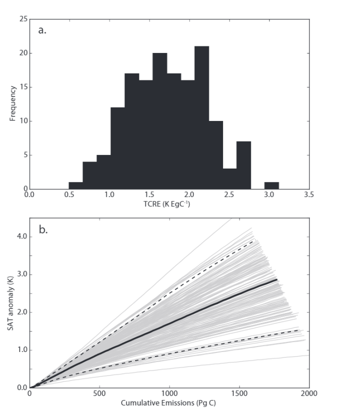
Approximating complex climate models I

- Matthews *et al* approximation

$$dT_t \approx \lambda \mathcal{E}_t dt$$

  where T_t is temperature and $\mathcal{E}_t$ is emissions.
- existing research studies the quality and time scale pertinent for this approximation.

Carbon Climate Response?



(a) Histogram of temperature responses to emission from physics ensemble experiment. (b) Cumulative emissions versus temperature curves solid black line is the mean, and dashed lines are the 5th and 95th percentiles. Source: Figure 3 from MacDougall *et al* (2017).

Approximating complex climate models II

Emulation approximation:

- fit simplified time series models to output from complex climate models, e.g. Li and Jarvis (2009), Sanso and Forest (2009) and Moyer, Stein, and Sadowski (2018)

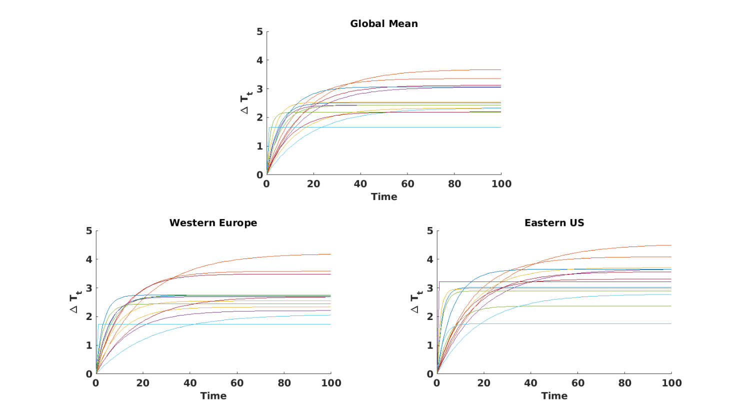
$$
\begin{aligned}
T_t &= \beta N_t + Z_t \\
dN_t &= -\rho N_t dt + \rho \log M_t dt \\
dZ_t &= -\phi(Z_t - \nu)dt + \sigma dW_t
\end{aligned}
$$

  where M_t is CO_2 concentration, Z_t captures transient dynamics, and dW_t is a Brownian increment
- for simplicity add:

$$
d \log M_t = \psi \left(\frac{E_t}{M_t} \right) dt
$$

  where M_t sums over alternative components of concentration

Climate model sensitivity



Temperature responses to a permanent doubling in CO2 concentration. Emulator approximations produced by Moyer, Stein and Sadowski

For within model parameter estimation using emulation, see *e.g.* Sanso and Forest (2009) and Olsen *et al* (2012)

Roadmap

Let’s assess inherent uncertainty in a dynamic economic model

- context: use a robust planner’s problem
- task: explore impact of alternative sources of uncertainty
    - temporary versus permanent shocks to productivity and investment
    - climate change shocks to technology
- target: what types of uncertainty have the biggest economic consequences?

Illustrative economic model with long-run risk

Model designed to have analytical solutions while embedding long run uncertainty. (Closely related to Eberly and Wang.)

- an AK technology that makes output be proportional to capital.
- output allocated between investment and consumption.
- adjustment costs

Capital evolves as

$$dK_t = .01 K_t \left[\mu_k(Z_t) - \vartheta_1 \frac{C_t}{K_t} dt - \frac{\vartheta_2}{2} \left(\frac{C_t}{K_t} \right)^2 dt + \sigma_k(Z_t) \cdot dW_t \right]$$

where K_t is the capital stock and C_t is consumption and Z_t is an exogenous forcing process that shifts investment opportunities.

Origins of financial market theory

Louis Bachelier (1900)
Brownian motion and efficient markets

Models of Asset Valuation

Two channels:

- Stochastic growth modeled as a process $G = \{G_t\}$ where G_t captures growth between dates zero and t.
- Stochastic discounting modeled as a process $S = \{S_t\}$ where S_t assigns risk-adjusted prices to cash flows at date t.

Date zero prices of a payoff G_t are

$$\pi = E\left(S_t G_t \mid \mathfrak{F}_0\right)$$

where $\mathfrak{F}_0$ captures initial period information.

Stochastic discounting reflects investor preferences through the intertemporal marginal rate of substitution for marginal investors. Two channels: exposure to uncertainty (quantity channel) and response to uncertainty (price channel).

Recursive valuation

- use a recursive specifications of preferences to highlight how uncertainty about future events affects asset valuation. Koopmans, Kreps and Porteus, Epstein and Zin, Bansal and Yaron
- Explore ways in which expectations and uncertainty about future growth rates influence the valuation of risky claims to consumption.

Beliefs about the future are reflected in current-period assessments through continuation values. The *forward-looking* nature of provides an additional channel through which *perceptions* about the future matter.

‘Risk’-Return Tradeoffs

Dynamic asset pricing through altering cash flow exposure to shocks.

- Study implication on the price today of changing the exposure tomorrow on a cash flow at some future date.
- Represent shock price elasticities by normalizing the exposure and studying the impact on the logarithms of the expected returns.
- Construct pricing counterpart to impulse response functions.

Elasticities

Counterparts to impulse response functions pertinent to valuation:

- shock-exposure elasticities
- shock-price elasticities

These are the ingredients to risk premia, and they have a term structure induced by the changes in the investment horizons.

Hansen-Scheinkman (*Finance and Stochastics*), Hansen (Fisher-Schultz, *Econometrica*), Borovička and Hansen ((*Journal of Econometrics*), Borovička-Hansen-Scheinkman (*Mathematical and Financial Economics*)

Shock elasticities formalism

- introduce martingale perturbations over small intervals of time.
- two dual interpretations
  - change the exposure of a cash flow to shocks
  - change the distribution of the shocks by including a local mean shifts
- compute implied changes in expected cash flows and in the expected returns

Mathematical tool: Malliavin calculus

Quantitative Example

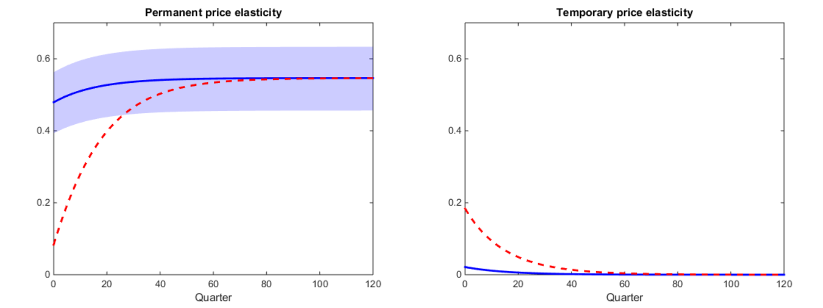
Bansal Yaron and long-run risk

$$dZ_t = -.021 Z_t dt + \sqrt{V_t} \begin{bmatrix} .031 & -.015 & 0 \end{bmatrix} dW_t$$
$$dV_t = -013 \left(V_t - 1 \right) dt + \sqrt{V_t} \begin{bmatrix} 0 & 0 & -.038 \end{bmatrix} dW_t$$
$$dY_t = (.01)(.15 + Z_t) dt + (.01) \sqrt{V_t} \begin{bmatrix} .34 & .7 & 0 \end{bmatrix} dW_t$$

- Y is the logarithm of consumption;
- Z captures predictability in macroeconomic growth rates;
- V captures stochastic volatility;
- Components of dW_t:

  permanent shock
  transitory shock
  stochastic volatility shock

Long-run risk shock-prices



Recursive utility and power utility. Bands depict .1 and .9 deciles.

Success?

- endows investors with knowledge of statistically subtle components of the macro time series. Where does this confidence come from?
- imposes stochastic volatility exogenously.
- requires large risk aversion.

Next steps

- incorporate climate change as a source of long-run uncertainty
- reinterpret large risk aversion as a modest concern for model misspecification - risk sensitive versus robust control
- include explicit model uncertainty

Alternative robustness concerns

Use a dynamic extension of max-min preferences. A decision maker considers alternative:

- structured models (allows for unknown parameters or parameters that drift)
- unstructured models that are statistically close (allows for misspecification)

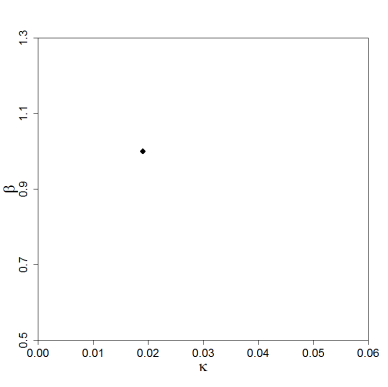
using relative entropy discrepancy.

Hansen and Sargent (The Price of Macroeconomic Uncertainty with Tenuous Beliefs) https://papers.ssrn.com/abstract=2888851

imports, refines and alters ideas from control theory and decision theory

Coefficient uncertainty

$$dY_t = .01\left(\alpha_y dt + \beta Z_t dt + \sigma_y \cdot dW_t\right) \qquad \text{macro evolution}$$
$$dZ_t = \alpha_z dt - \kappa Z_t dt + \sigma_z \cdot dW_t \qquad \text{growth evolution}$$



Sets of parameter values (β, κ) constrained by relative entropy.

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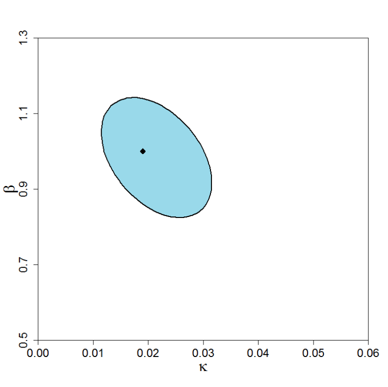

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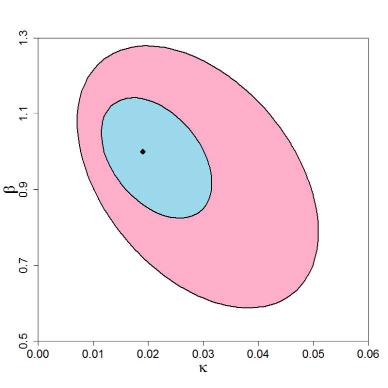


Sets of parameter values (β, κ) constrained by relative entropy.

Misspecified Dynamics

- represent alternative structured models as drift distortions R_t:

$$dW_t = R_t dt + dW_t^R$$

- introduce unstructured drift distortions U_t:

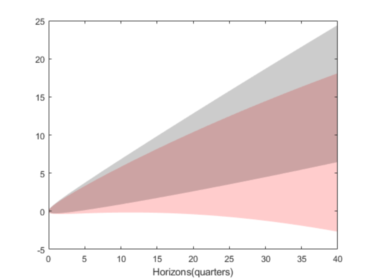
$$dW_t = (U_t - R_t)dt + dW_t^{U-R}$$

- impose a quadratic penalty $\frac{1}{2}|U_t - R_t|^2$ where R_t is one of the possible modeled drift distortions
- minimize over the restricted family of R_t's and over the less restricted family U_t subject to penalization

Special case of variational preferences and extends Hansen-Sargent (AER).

Adjustment for uncertainty

Slant probabilities in a cautious direction! Locate the probability model that is most consequential to the decision maker. Contributes to the stochastic discount factor used in marginal valuation.



The **grey** shaded region gives the .1 and .9 deciles for the baseline model. The red shaded region gives the .1 and .9 deciles.

Climate impacts

Incorporating climate impacts into the analysis.

- temperature shifts proportionately consumption and capital by a multiplicative factor $\exp(-\gamma T_t)$ where T_t is temperature relative to some preindustrial initial state.
- limit emissions with a Hotelling finite stock constraint
- instantaneous utility contribution:

$$(1-\alpha)\log C_t + \alpha \log \mathcal{E}_t$$

- incorporate uncertainty in the emissions to temperature evolution

Asset pricing tools for policy analysis

direct application of marginal analysis

- market valuation
- local policy analysis
- Pigouvian taxation

indirect benefit: tractable starting point for discrete policy assessment

Pigouvian tax illustration

One measure of the social cost of carbon is the optimal Pigouvian tax:

$$\left[\left(\lambda + \frac{1}{\kappa}|\sigma_\tau|^2\right)\gamma(1-\alpha) + \frac{d\upsilon}{dr}(r)\right]\frac{(1-\alpha)c^* K_t \exp(-\gamma T_t)}{\alpha \mathcal{E}_t}.$$

- first term in the square brackets captures the dynamic contribution of emissions on temperature
- second term captures the contribution of emissions on the resource stock depletion

Robustness implications

Pigouvian tax:

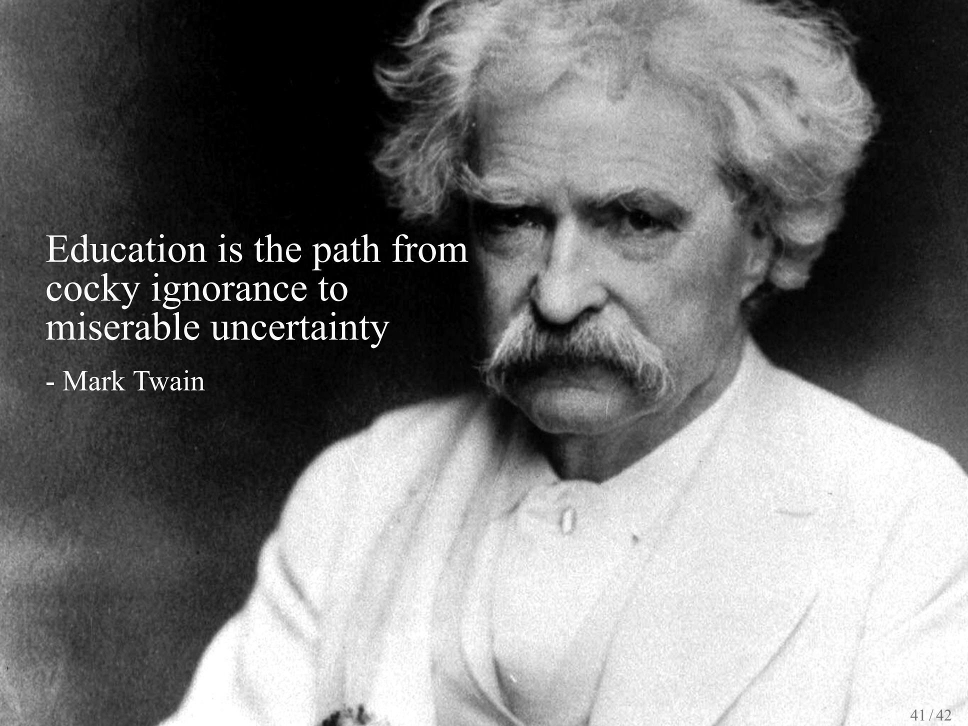
$$\left[\left(\lambda + \frac{1}{\kappa}|\sigma_\tau|^2\right)\gamma(1-\alpha) + \frac{d\upsilon}{dr}(r)\right]\frac{(1-\alpha)c^* K_t \exp(-\gamma T_t)}{\alpha \mathcal{E}_t}.$$

- robustness considerations augment the presumed long-term impact of emissions on temperature through the parameter κ (Brock and Hansen, https://ssrn.com/abstract=3008833)
- in a related analysis with damage uncertainty (uncertainty about γ), higher greenhouse gas concentration alters the cost of model misspecification (Li, Loch-Temzelides, Narajabad, *Quantitative Economics*, 2016)

Uncertainty and climate change policy

"Any serious discussion of the changing climate must begin by *acknowledging* not only the scientific certainties but also the *uncertainties*, especially in projecting the future. Recognizing those limits, rather than ignoring them, will lead to a more *sober* and ultimately more *productive* discussion of climate change and climate policies."

Steven E. Koonin (2014, former undersecretary for science in the US Department of Energy)



Education is the path from cocky ignorance to miserable uncertainty

- Mark Twain

Conclusion

Asset pricing provides valuable tools for policy analysis in the presence of uncertainty!

- market valuation
- local policy analysis
- Pigouvian taxation

indirect benefit: tractable starting point for discrete policy assessment