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Introduction to model uncertainty and robustness

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Abstract

This article introduces the symposium on model uncertainty and robustness.

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This symposium contains four papers about decision making under uncertainty in contexts in which a decision maker cannot or does not formulate a single probability model. Ways of thinking about this problem share common features owing to Gliboa and Schmeidler [1], who axiomatized a max–min expected utility theory in which a decision maker chooses an action to maximize an expected utility function whose probability weights he supposes to be chosen by a malevolent agent. Especially in dynamic settings, we continue to learn how to formulate the set of models over which this malevolent agent chooses.

Maccheroni, Marinacci, and Rustichini [3] extend to multi-period problems the static theory of variational preferences that they introduced in an earlier paper. A preference is called variational if the decision maker chooses an act while a malevolent opponent (Nature) chooses the probability over states at some cost. Variational preferences include Multiple Priors preferences, Robust Control preferences, and Mean-Variance preferences. In the extension to many periods, both Nature and the decision maker choose in every period. The paper aims to characterize time

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consistent preferences. The main result is that variational preferences are time consistent if and only if their cost functions satisfy a generalization of Bayes' rule and a 'no gain' condition. Thanks to this characterization, to decide whether a variational preference is time consistent it is enough to check whether its cost function satisfies these two conditions. Robust Control preferences are time consistent, while Mean-Variance preferences are not.

All formulations of decision making in the face of ambiguity about model specification have the decision maker carry along a set of models. In any applied problem, a researcher must decide how to specify that set of models. It is remarkable that, without appealing to any axiomatic foundations, robust control theorists have used max–min decision theory as a systematic way of studying the fragility of dynamic policy rules to model misspecifications. Hansen, Sargent, Turmuhambetova, and Williams [2] view robust control theory as a promising way to specify the set of models in the theory of Gilboa and Schmeidler [1] by taking as a benchmark a *single* model that is regarded as the decision maker's approximating model, one that he does not fully trust, and surrounding it with a cloud of models that are close as measured by relative entropy. The paper describes the relationships among alternative mathematical formalizations of continuous time stochastic decision problems that are designed to produce robust policy functions. The formalizations differ in terms of the probabilities spaces on which they are defined and how sets of alternative models are characterized. The paper describes a set of observational equivalence relations among these alternative formulations. In addition, it establishes a connection between a stochastic differential utility model without fear of model misspecification and a related model of a decision maker who fears model misspecification and that is observationally equivalent with respect to particular experiments.

Several models of choice under ambiguity are characterized by sets of probabilities. It is convenient to interpret such probabilities as alternative, equally plausible descriptions of the underlying uncertainty. In turn, this enables one to associate specific decision models with distinct attitudes towards ambiguity (e.g., 'max–min–expected utility' reflects pessimism, etc.). However, the same preferences typically admit several distinct representations, each featuring a different set of probabilities and a different decision model. For example, generically, max–min–expected utility preferences a la Gilboa–Schmeidler also admit a continuum of alpha-maxmin representations, each associated with a different sets of priors. Siniscalchi [5] proposes a formal criterion that identifies probabilities independently of the individual's decision model, and hence captures the sprit of the intuitive interpretation of multiple priors in a robust way. The paper axiomatizes the class of preferences characterized by priors that are 'plausible' according to the proposed definition, and relates it to known decision models.

Maenhout [4] incorporates a concern about robustness, or model-uncertainty aversion, in a dynamic portfolio choice problem with stochastic investment opportunities (non-i.i.d. returns because of a mean-reverting risk premium). The closed-form solutions demonstrate that robustness reduces the optimal equity weight in the portfolio, but increases the relative importance of the intertemporal hedging demand. The paper also presents a new methodology for calculating detection-error probabilities in continuous-time models. The methodology is based on Fourier inversion of the conditional characteristic functions of the Radon–Nikodym derivatives between the reference model and the alternative model considered by the robust decision maker, and can be applied in jump-diffusion environments.

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