Approximate Models
and
Robust Decisions

Lars Peter Hansen
University of Chicago
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Probability meets Social Science

Jacob Bernoulli (left)

Law of Large Numbers: how unknown probabilities are revealed (1713)

Dual Roles for Statistics in Economic Analysis

- Outside a model
  Given a dynamic economic model, researchers:
  - estimate unknown parameters
  - assess model implications

- Inside a model
  When *constructing* a dynamic economic model, researchers:
  - depict economic actors (consumers, enterprises) as they cope with uncertainty
  - deduce the consequences for market outcomes and resource allocations

Placing Uncertain Investors Inside an Economic Model

When *constructing* a dynamic economic model, researchers:

- ▷ depict economic actors (consumers, enterprises) as they cope with uncertainty when making economic decisions with future consequences
- ▷ deduce the resulting market responses and consequences for resource allocations

Rational Expectations inside an Economic Model

Muth (1961) and Lucas (1972): Economic actors (investors) use *long histories* of data to infer the model, including its parameters.

- ▷ Yields a stochastic notion of *equilibrium* with expectations determined *inside the model*
- ▷ Gives a coherent approach to *policy analysis*

Influential, but *neglects* some components of uncertainty by featuring only *risk*. Statistical challenges are off the table.

Risk Inside the Model

- Recent empirical successes rely on endowing investors with knowledge of statistically subtle components of the macro time series. Where does this confidence come from?
- Imposes stochastic volatility exogenously.
- Imposes large risk aversion.

Success?

Uncertainty in the Macroeconomy

- IMF report:

  The Brexit vote implies a substantial increase in economic, political, and institutional uncertainty, which is projected to have negative macroeconomic consequences, especially in advanced European economies.

- Obstfeld (IMF chief economist):

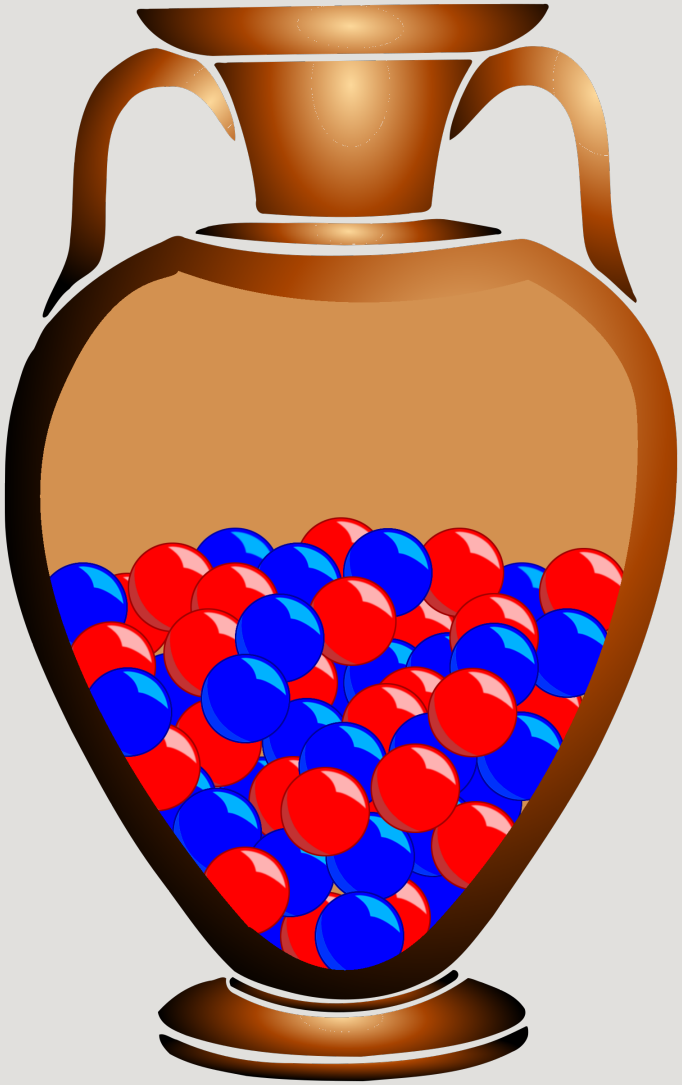
  The real effects of Brexit will play out gradually over time, adding elements of economic and political uncertainty that could be resolved only after many months. This overlay of extra uncertainty, in turn, may open the door to an amplified response of financial markets to negative shocks.

Statistical complexity

- When is it challenging to learn and draw inferences?
- When is there more scope for behavioral distortions?
- When could statistical uncertainty induce fluctuations in prices of uncertainty that are observed in financial markets?

Take a broader perspective on uncertainty that is typical in economic analyses.

Uncertainty can be *risk*



50 Red Balls

50 Blue Balls

Uncertainty can be *ambiguity*

? Red Balls

? Blue Balls

Uncertainty can *change over time*

? Red Balls

? Blue Balls

Multiple Components to Uncertainty

- Model *risk* - what probabilities does a model assign to events in the future?

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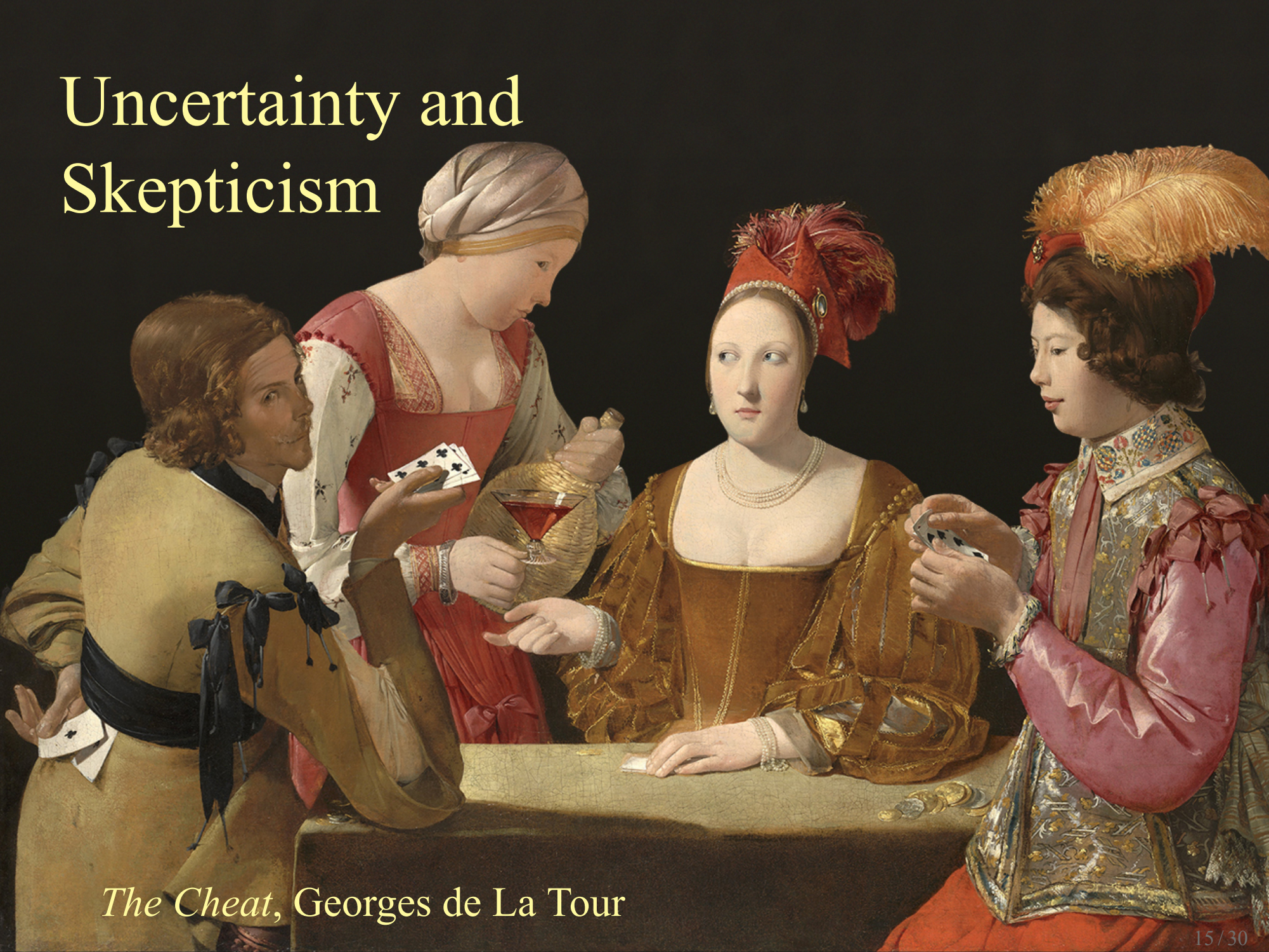
- Model *ambiguity* - how much confidence do we place in each model?
- Model *misspecification* - how do we use models that are not perfect?

Uncertainty and Skepticism



The Cheat, Georges de La Tour

Advances in Decision Theory under Uncertainty

- Initial contributions from statistics: Wald, De Finetti, Savage, Good and others on decision theory and subjective probabilities.
- Initial contributions from economics: Koopmans, Kreps and Porteus and others on the intertemporal composition of risk.
- Robust Bayesian analysis
- Robust control theory
- More recent contributions of economics literature - axiomatic based analyses of ambiguity aversion in decision making.

Gives researchers structured ways to explore the impact of inside the model uncertainty and gives statisticians a way to frame evidence in providing support to policy makers.

Model Misspecification and Ambiguity Aversion

Statistical models we use in practice are misspecified.

Aim of robust approaches:

- ▷ use models in sensible ways rather than discard them
- ▷ use probability and statistics to provide tools for assessing sensitivity to potential misspecification

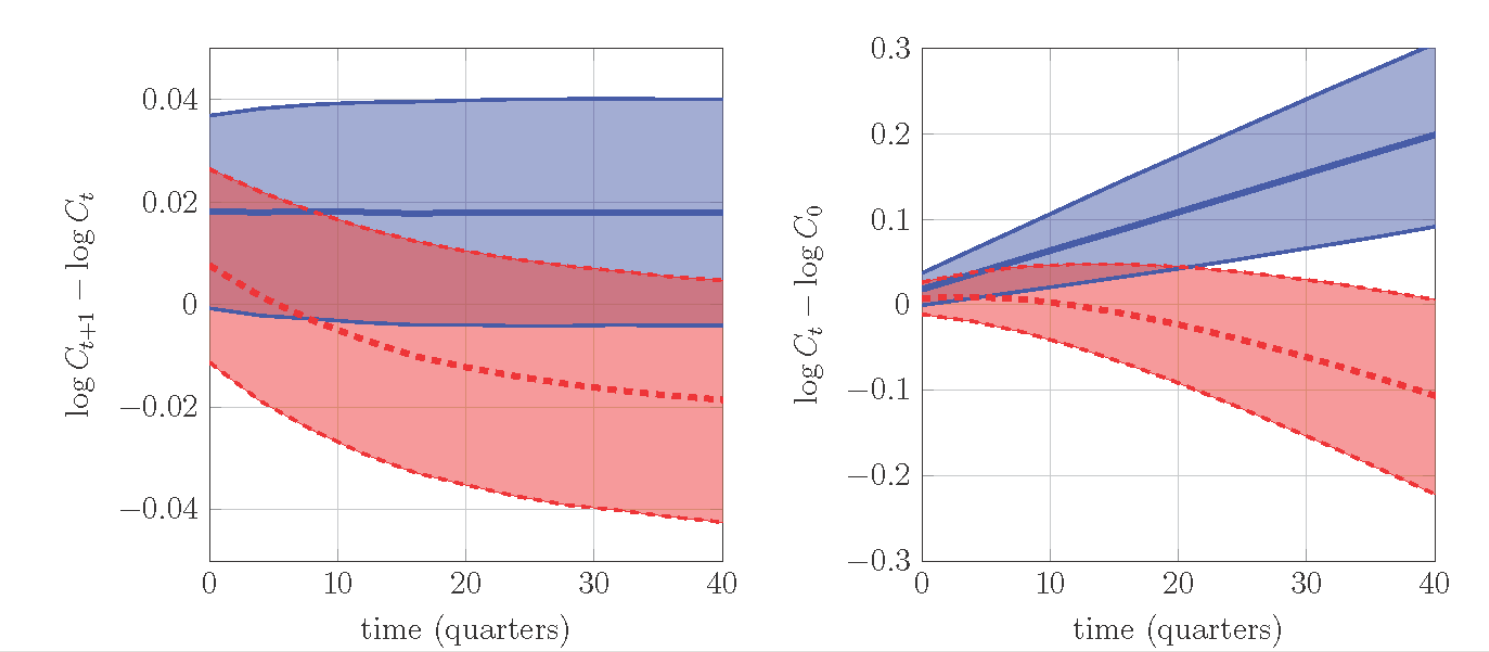
Ambiguity aversion (in contrast to risk aversion) targets the forms of misspecification with the most adverse consequences for the decision maker.

Long-term Uncertainty

Concerns about model misspecification in dynamic settings often feature long-term uncertainty

- Asset pricing evidence suggest that long-term "risk adjustments" are prominent in asset prices (Alvarez and Jermann)
- Depict Investor preferences using recursive utility specification with a concern for the intertemporal composition of risk (Epstein and Zin and Bansal and Yaron)
- Reinterpret by expressing investor concerns about model misspecification (Hansen and Sargent)
- Represent long-term uncertainty prices using a long-term risk neutral probability measure characterized by Hansen and Schienkman and Borovicka, Hansen and Scheinkman.

Market Adjustments for Long-term Uncertainty



Blue are the actual interquartile range of the distribution and red includes market adjustments for long-term uncertainty. Source: Borovicka, Hansen and Scheinkman.

Macroeconomics and asset pricing

Model the investment in risky capital and the pricing of financial assets:

$$E\left[\left(\frac{S_{t+\ell}}{S_t}\right)X_{t+\ell}\middle|\mathcal{F}_t\right]=Q_t$$

where

- S is a stochastic discount factor process
- $X_{t+\ell}$ vector of payoffs on physical or financial assets
- ℓ is the investment horizon
- $\mathcal{F}_t$ is the investor information
- Q_t vector of asset prices

Parametric Implementation

Model the investment in risky capital and the pricing of financial assets:

$$E\left[\left(\frac{S_{t+\ell}}{S_t}\right)X_{t+\ell}\middle|\mathcal{F}_t\right]=Q_t$$

- Estimation entails parameterizing S:

$$\left(\frac{S_{t+\ell}}{S_t}\right)=\phi(Y_{t+\ell},\beta)$$

  where β is an unknown parameter to be estimated.
- Estimation avoids specifying fully investor information.

In most applied research E is the expectation implied by the data generating process and used by investors *inside the model*.

Nonparametric Implementation

Model the investment in risky capital and the pricing of financial assets:

$$E\left[\left(\frac{S_{t+\ell}}{S_t}\right)X_{t+\ell}\middle|\mathcal{F}_t\right]=Q_t$$

- Use asset market data alone to "infer" the properties of the stochastic discount factor process. Opportunity to exploit rich data sets.
- With incomplete information from asset market, the empirical pricing restrictions are consistent with many SDF's. Characterize the properties of the solutions.
- Explore dynamic structure: what makes the "price" of uncertainty fluctuate as reflected in financial markets?

Asset Pricing under Subjective Beliefs

$$\widetilde{E}\left[\left(\frac{\widetilde{S}_{t+\ell}}{\widetilde{S}_t}\right) X_{t+\ell} \middle| \mathcal{F}_t\right] = Q_t \tag{1}$$

where $\widetilde{E}$ is the subjective expectation operator and $\widetilde{S}$ is the corresponding stochastic discount factor.

- Represent belief distortions using a positive martingale M:

$$\widetilde{E}\left[Y_{t+\ell} | \mathcal{F}_t\right] = E\left[\left(\frac{M_{t+\ell}}{M_t}\right) Y_{t+\ell} \middle| \mathcal{F}_t\right]$$

  M is a likelihood ratio which allows for a statistical perspective.
- Then $S = M\widetilde{S}$.
- Ambiguity aversion targeted to model misspecification gives a specific model for M.

Push Back from Rational Expectations

Stochastic discount factor decomposition

$$S = \underset{\substack{\text{subjective}\\ \text{beliefs}}}{M} \times \underset{\substack{\text{risk}\\ \text{preferences}}}{\widetilde{S}}$$

- ▷ Even rich data from financial markets identifies only S but not the product.
- ▷ Impose parametric structure on one or both components
  - Decision maker confronts potential model misspecification under ambiguity aversion. Identifies a particular "belief distortion". Uncertainty "inside the model."
  - Econometrician confronts an identification failure and characterizes a large family of potential belief distortions. Uncertainty "outside the model."

Statistical Complexity and Misspecification

- Introduce a family of positive martingales $\{M_t : t \geq 0\}$, $M_0 = 1$ to characterize alternative potential models in a dynamic setting.
- Introduce a statistical discrepancy measure called relative entropy:

$$\lim_{t\to\infty} \frac{1}{t} E\left(M_t \log M_t | \mathcal{F}_0\right)$$

$$= \lim_{t\to\infty} \sum_{u=0}^{t-1} E\left[M_{u+1}\left(\log M_{u+1} - \log M_u\right)|\mathcal{F}_0\right]$$

$$= \lim_{\delta\downarrow 0}[1-\exp(-\delta)] \sum_{u=0}^{\infty} E\left[\exp(-u\delta) M_{u+1}\left(\log M_{u+1} - \log M_u\right)|\mathcal{F}_0\right]$$

- Explore consequences of model misspecification captured by alternative positive martingales and subject to relative entropy penalization.

Three Related a Applications

- Dynamic decision or control problems: robust control. What forms of misspecification should a decision maker fear the most?
- Dynamic stochastic equilibrium models: prices of uncertainty. What prices that are revealed in financial markets when investors confront uncertainty?
- Econometric analyses: alternative to GMM estimation. How large statistically do belief distortions have to be to explain the data?

No separation between decisions and statistical inferences.

Friedrich Hayek (1974)

"Even if true scientists should recognize the limits of studying human behaviour, as long as the public has expectations, there will be people who *pretend* or *believe* that they can do more to meet popular demand than what is really in their power."
(From Hayek's Nobel address)

Macroeconomic Policy

Milton Friedman

"As Josh Billings wrote many years ago, 'The trouble with most folks isn't so much their ignorance, as knowing so many things that ain't so.' Pertinent as this remark is to economics in general, it is especially so in monetary economics." (1965)

Kenneth Rogoff

"It's wrong to vilify the Fed for hiking, and it's wrong to vilify it for not hiking; if it is such a close call, it probably doesn't matter so much. But, at this critical point, it is fair to ask the Fed for a much clearer message about what its strategy is, and what this implies for the future." (2015)

Uncertainty and Climate Change Policy

"Any serious discussion of the changing climate must begin by *acknowledging* not only the scientific certainties but also the *uncertainties*, especially in projecting the future. Recognizing those limits, rather than ignoring them, will lead to a more *sober* and ultimately more *productive* discussion of climate change and climate policies."

Steven E. Koonin (2014, former undersecretary for science in the US Department of Energy)

Do complicated problems require complicated solutions?

Financial market oversight is arguably a complicated problem

- Acknowledge limits to our understanding of the linkages between financial markets and the macroeconomy.
- Specific models could imply alternative complex solutions.
- "Robust" policies perform well across alternative models.
- Simple robust policies avoid adding uncertainty to the economic environment.