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## Intangible Risk

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### 4.1 Introduction

Accounting for the asset values by measured physical capital and other inputs arguably omits *intangible* sources of capital. This intangible or unmeasured component of the capital stock may result because some investments from accounting flow measures are not eventually embodied in the physical capital stock. Instead there may be scope for valuing ownership of a technology, for productivity enhancements induced by research and development, for firm-specific human capital, or for organizational capital.

For an econometrician, intangible capital becomes a residual needed to account for values. In contrast to measurement error, omitted information, or even model approximation error, this residual seems most fruitfully captured by an explicit economic model. It is conceived as an input into technology whose magnitude is not directly observed. Its importance is sometimes based on computing a residual contribution to production after all other measured inputs are accounted for. Alternatively it is inferred by comparing asset values from security market data to values of physical measures of firm or market capital. Asset market data is often an important ingredient in the measurement of intangible capital. Asset returns are used

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to convey information about the marginal product of capital and asset values are used to infer magnitude of intangible capital.

In the absence of uncertainty, appropriately constructed investment returns should be equated. With an omitted capital input, constructed investment returns across firms, sectors, or enterprises will be heterogeneous because of mis-measurement. As argued by Telser (1988) and many others, differences in measured physical returns may be “explained by the omission of certain components of their ‘true’ capital.” McGrattan and Prescott (2000) and Atkeson and Kehoe (2002) are recent macroeconomic examples of this approach. Similarly, as emphasized by Hall (remarks in this volume) and Lev and Radhakrishnam (chap. 3 in this volume), asset values should encode the values of both tangible and of intangible capital. Thus, provided that physical capital can be measured directly, security market data should convey information about the intangible component of the capital stock. This rationale provides the basis for the cross-sectional approaches to measurement discussed by Brynjolfsson and Hitt (2004) and references therein.

Following Hall (2001), we find it fruitful to consider the impact of risk in the measurement of intangible capital. Although not emphasized by Hall, there is well documented heterogeneity in the returns to equity of different types. In the presence of uncertainty, it is well known that use of a benchmark asset return must be accompanied by a risk adjustment. Historical averages of equity returns suggest that the riskiness of tangible investment differs systematically from the riskiness of intangible investment. Asset return and price heterogeneity can be caused not only by differences in the marginal products but also by differences in the relative values of capital across types. Even under the presumption that markets value risk correctly, inferences about the intangible capital stock using security market prices must necessarily confront these secular movements in relative valuation. Unfortunately, a recurring problem in financial economics is the construction of a valuation model that is consistent with observed return heterogeneity.

In section 4.2 we develop a model of investment and examine alternative assumptions used in the literature to make inferences about the intangible component of capital based on market valuations. We show that these assumptions typically impose a restriction on asset returns and/or relative prices. In section 4.3 we review and reproduce some of the findings in the asset pricing literature by Fama and French (1992) on return heterogeneity that may be linked to differences in intangible investment across firms. Risk premia can be characterized in terms of return risk or dividend or cash flow risk. We follow some recent literature in finance by exploring dividend risk. Since equity ownership of securities entitles an investor to future claims to dividends in all subsequent time periods, quantifying divi-

dividend risk requires a time series process. We consider measurements of dividend risk using vector autoregressive (VAR) characterizations. Since asset valuation entails the study of a present-value relation, long-run growth components of dividends can play an important role in determining asset values. In section 4.4 we reproduce the present-value approximation used in the asset pricing literature and use it to define a long-run measure of risk as a discounted impulse response. In sections 4.5 and 4.6 we use VAR methods to estimate the dividend-risk measures that have been advocated in the asset-pricing literature.

The literatures on intangible capital and asset return heterogeneity to date have been largely distinct. Our disparate discussion of these literatures will inherit some of this separation. In section 4.7 we conclude with discussion of how to understand better lessons from asset pricing for the measurement of intangible capital.

## 4.2 Adjustment Cost Model

We begin with a discussion of adjustment costs and physical returns. Grunfeld (1960) shows how the market value of a firm is valuable in the explanation of corporate investment. Lucas and Prescott (1971) developed this point more fully by producing an equilibrium model of investment under uncertainty. Hayashi (1982) emphasized the simplicity that comes with assuming constant returns to scale. We exploit this simplicity in our development that follows.

Consider the following setup.

### 4.2.1 Production

Let  $n_t$  denote a variable input into production such as labor, and suppose there are two types of capital, namely  $k_t = (k_t^m, k_t^u)$  where  $k_t^m$  is the measured capital and  $k_t^u$  is unmeasured or intangible capital stock. Firm production is given by

$$f(k_t, n_t, z_t),$$

where  $f$  displays constant returns to scale in the vector of capital stocks and the labor input  $n_t$ . The random variable  $z_t$  is a technology shock at date  $t$ .

Following the adjustment cost literature, there is a nonlinear evolution for how investment is converted into capital.

$$(1) \quad k_{t+1} = g(i_t, k_t, x_t),$$

where  $g$  is a two-dimensional function displaying constant returns to scale in investment and capital and  $x_t$  is a specific shock to the investment technology. We assume that there are two components of investment corresponding to the two types of capital. This technology may be separable, in

which case the first coordinate of  $g$  depends only on  $i_t^m$  and  $k_t^m$  while the second coordinate depends only on  $i_t^u$  and  $k_t^u$ .

EXAMPLE 1. *A typical example of the first equation in system (1) is*

$$k_{t+1}^m = (1 - \delta_m)k_t^m + i_t^m - g_m\left(\frac{i_t^m}{k_t^m}, x_t\right)k_t^m,$$

where  $\delta_m$  is the depreciation rate and  $g_m$  measures the investment lost in making new capital productive.

In the absence of adjustment costs, the function  $g$  is linear and separable.

EXAMPLE 2. *A common specification that abstracts from adjustment costs is*

$$g(k_t, i_t, x_t) = \begin{bmatrix} 1 - \delta_m & 0 \\ 0 & 1 - \delta_u \end{bmatrix} k_t + i_t.$$

#### 4.2.2 Firm Value

Each time period the firm purchases investment goods and produces. Let  $p_t$  denote the vector of investment good prices and  $w_t$  the wage rate. Output is the numeraire in each date. The date-zero firm value is

$$E\left\{\sum_{t=0}^{\infty} S_{t,0}[f(k_t, n_t, z_t) - p_t \cdot i_t - w_t n_t] \mid F_0\right\}.$$

The firm uses market-determined stochastic discount factors to value cash flows. Thus,  $S_{t,0}$  discounts the date- $t$  cash flow back to date zero. This discount factor is stochastic and varies depending on the realized state of the world at date  $t$ . As a consequence  $S_{t,0}$  not only discounts known cash flows but adjusts for risk; see Harrison and Kreps (1979) and Hansen and Richard (1987).<sup>1</sup> The notation  $F_0$  denotes the information available to the firm at date zero.

Form the Lagrangian

$$E\left\{\sum_{t=0}^{\infty} S_{t,0}[f(k_t, n_t, z_t) - p_t \cdot i_t - w_t n_t - \lambda_t \cdot [k_{t+1} - g(i_t, k_t, x_t)]] \mid F_0\right\},$$

where  $k_0$  is a given initial condition for the capital stock. First-order conditions give rise to empirical relations and valuation relations that have been used previously.

Consider the first-order conditions for investment:

$$(2) \quad p_t = \frac{\partial g}{\partial i}(i_t, k_t, x_t)' \lambda_t.$$

1. This depiction of valuation can be thought of assigning state prices, but it also permits certain forms of market incompleteness.

Special cases of this relation give rise to the so-called  $q$  theory of investment. Consider, for instance, the separable specification in Example 1. Then

$$(3) \quad \frac{\partial g_m \left( \frac{i_t^m}{k_t^m} \right)}{\partial i} x_t = 1 - \frac{p_t^m}{\lambda_t^m}.$$

This relates the investment capital ratio to what is called Tobin's  $q$  ( $q_t = \lambda_t^m / p_t^m$ ). The Lagrange multiplier  $\lambda_t^m$  is the date- $t$  shadow value of the measured capital stock that is productive at date  $t$ . There is an extensive empirical literature that has used equation (3) to study the determinants of investment. As is well known,  $\lambda_t^m = p_t^m$  and Tobin's  $q$  is equal to one in the absence of adjustment costs as in Example 2.

Consider next the first-order condition for capital at date  $t + 1$ :

$$\lambda_t = E \left\{ S_{t+1,t} \left[ \frac{\partial f}{\partial k}(k_{t+1}, n_{t+1}, z_{t+1}) + \frac{\partial g}{\partial k}(i_{t+1}, k_{t+1}, x_{t+1})' \lambda_{t+1} \right] \middle| F_t \right\},$$

where  $S_{t+1,t} \equiv S_{t+1,0} / S_{t,0}$  is the implied one-period stochastic discount factor between dates  $t$  and  $t + 1$ . This depiction of the first-order conditions is in the form of a one-period pricing relation. As a consequence, the implied returns to investments in the capital goods are

$$r_{t+1}^m \equiv \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1}) + \frac{\partial g}{\partial k^m}(i_{t+1}, k_{t+1}, x_{t+1})' \lambda_{t+1}}{\lambda_t^m}$$

$$r_{t+1}^u \equiv \frac{\frac{\partial f}{\partial k^u}(k_{t+1}, n_{t+1}, z_{t+1}) + \frac{\partial g}{\partial k^u}(i_{t+1}, k_{t+1}, x_{t+1})' \lambda_{t+1}}{\lambda_t^u}.$$

The denominators of these shadow returns are the marginal costs to investing an additional unit capital at date  $t$ . The numerators are the corresponding marginal benefits reflected in the marginal product of capital and the marginal contribution to productive capital in future time periods. The shadow returns are model-based constructs and are not necessarily the same as the market returns to stock or bond holders.

In the separable case (example 1), the return to the measurable component of capital is

$$r_{t+1}^m = \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1}) k_{t+1}^m + \lambda_{t+1}^m k_{t+2}^m - p_{t+1}^m i_{t+1}^m}{\lambda_t^m k_{t+1}^m}.$$

An alternative depiction can be obtained by using the investment first-order conditions to substitute for  $\lambda_t^m$  and  $\lambda_{t+1}^m$  as in Cochrane (1991b). In the absence of adjustment costs (example 2), the return to tangible capital is

$$(4) \quad r_{t+1}^m = \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1}) + (1 - \delta_m)p_{t+1}^m}{p_t^m}.$$

The standard stochastic growth model is known to produce too little variability in physical returns relative to security market counterparts. In the one-sector version, the relative price  $p_t^m$  becomes unity. As can be seen in equation (4), the only source of variability is the marginal product of capital. Inducing variability in this term by variability in the technology shock process  $z_{t+1}$  generates aggregate quantities such as output and consumption that are too variable.

The supply of capital is less elastic when adjustment costs exist; hence, models with adjustment costs can deliver larger return variability than the standard stochastic growth model. This motivated Cochrane (1991b) and Jermann (1998) to include adjustment costs to physical capital in their attempts to generate interesting asset market implications in models of aggregate fluctuations. As an alternative, Boldrin, Christiano, and Fisher (2001) study a two-sector model with limited mobility of capital across technologies. In our environment, limited mobility between physical and intangible capital could be an alternative source of aggregate return variability.

When we restrict the technology to be constant returns to scale, the time zero firm value is

$$(5) \quad f(k_0, n_0, z_0) - i_0 \cdot p_0 - w_0 n_0 + k_1 \cdot \lambda_0 \\ = k_0 \cdot \left[ \frac{\partial f}{\partial k}(k_0, n_0, z_0) + \frac{\partial g}{\partial k}(i_0, k_0, x_0)' \lambda_0 \right].$$

This relation is replicated over time. Thus the date- $t$  firm value is given by the cash flow (profit) plus the *ex-dividend* price of the firm. Equivalently, it is the value of the date-zero vector of capital stocks taking account of the marginal contribution of this capital to the production of output and to capital in subsequent time periods. Thus, asset market values can be used to impute  $k_{t+1} \cdot \lambda_t$  after adjusting for firm cash flow. When the firm has unmeasured *intangible* capital, this additional capital is reflected in the asset valuation of the firm. Even if the tangible component of the capital ( $k_{t+1}^m$ ) can be measured, variation in the relative price of intangible capital makes it difficult to infer a physical measure of  $k_{t+1}^u$ .

The presence of intangible capital alters how we interpret Tobin's  $q$ . In effect, there are now multiple components to the capital stock. Tobin's  $q$  is typically measured as a ratio of values and not as a simple ratio of prices. While the market value of a firm has both contributions, a replacement value constructed by multiplying the price of new investment goods by the

measured capital stock will no longer be a simple price ratio. Instead, we would construct

$$\frac{\lambda_t \cdot k_{t+1}}{p_t^m k_{t+1}^m}.$$

Heterogeneity in  $q$  across firms or groups of firms reflects in part different amounts of intangible capital, not simply a price signal to convey the profitability of investment.

The dynamics of the ex-dividend price of the firm are given by

$$\lambda_t \cdot k_{t+1} = E\{S_{t+1,t}[f(k_{t+1}, n_{t+1}, z_{t+1}) - p_{t+1} \cdot i_{t+1} - w_{t+1}n_{t+1} + k_{t+2} \cdot \lambda_{t+1}] | F_t\}.$$

The composite return to the firm is thus

$$(6) \quad r_{t+1}^c = \frac{f(k_{t+1}, n_{t+1}, z_{t+1}) - p_{t+1} \cdot i_{t+1} - w_{t+1}n_{t+1} + k_{t+2} \cdot \lambda_{t+1}}{\lambda_t \cdot k_{t+1}} \\ = \frac{\lambda_t^m k_{t+1}^m r_{t+1}^m + \lambda_t^u k_{t+1}^u r_{t+1}^u}{\lambda_t \cdot k_{t+1}}.$$

Recall that  $k_{t+1}$  is determined at date  $t$  (but not its productivity) under our timing convention. The composite return is a weighted average of the returns to the two types of capital with weights given by the relative values of the two capital stocks.

Firm ownership includes both bond and stock holders. The market counterpart to the composite return is a weighted average of the returns to the bond holders and equity holders with portfolio weights dictated by the amount of debt and equity of the firm.

#### 4.2.3 Imputing the Intangible Capital Stock

These valuation formulas have been used by others to make inferences about the intangible capital stock. First we consider a return-based approach. We then consider a second approach based on asset values.

Following Atkeson and Kehoe (2002) and others, we exploit the homogeneity of the production function and Euler's Theorem to write

$$y_{t+1} = \frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^m + \frac{\partial f}{\partial n}(k_{t+1}, n_{t+1}, z_{t+1})n_{t+1} \\ + \frac{\partial f}{\partial k^u}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^u,$$

where  $y_{t+1} = f(k_{t+1}, n_{t+1}, z_{t+1})$  is output. Thus, the contribution of intangible capital to output is measured by

$$\frac{\frac{\partial f}{\partial k^u}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^u}{y_{t+1}} = 1 - \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^m}{y_{t+1}} - \frac{\frac{\partial f}{\partial n}(k_{t+1}, n_{t+1}, z_{t+1})n_{t+1}}{y_{t+1}}.$$

To make this operational we require a measure of the labor share of output given by compensation data and a measure of the share of output attributed to measured component of capital. Using formula (4) from example 2 and knowledge of the return and the depreciation rate, we can construct

$$\frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})}{p_t^m} = r_{t+1}^m - (1 - \delta_m) \frac{p_{t+1}^m}{p_t^m}.$$

Thus,

$$(7) \quad \frac{\frac{\partial f}{\partial k^m}(k_{t+1}, n_{t+1}, z_{t+1})k_{t+1}^m}{y_{t+1}} = \left[ r_{t+1}^m - (1 - \delta_m) \frac{p_{t+1}^m}{p_t^m} \right] \frac{p_t^m k_{t+1}^m}{y_{t+1}}.$$

This formula avoids the need to directly measure rental income to measured capital, but it instead requires measures of the physical return, physical depreciation scaled by value appreciation, and the relative value of tangible capital to income.

The physical return to measured capital is not directly observed. Even if we observed the firm's (or industry's or aggregate) return from security markets, this would be the composite return in equation (6) and would include the contribution to intangible capital. As a result, a time series of return data from security markets is not directly usable. Instead Atkeson and Kehoe (2002) take a steady-state approximation, implying that returns should be equated to measure the importance of intangible capital in manufacturing. Income shares and price appreciation are measured using time series averages. Given the observed heterogeneity in average returns, as elsewhere in empirical studies based on the deterministic growth model, there is considerable ambiguity as to which average return to use. To their credit, Atkeson and Kehoe document the sensitivity of their intangible capital measure to the assumed magnitude of the return.<sup>2</sup> We will have more to say about return heterogeneity subsequently.

2. Atkeson and Kehoe (2002) are more ambitious than what we describe. They consider some tax implications and two forms of measured capital: equipment and structures. Primarily they develop and apply an interesting and tractable model of organizational capital.



To infer the value of the intangible capital relative to output using return data, we combine equation (7) with its counterpart for intangible capital to deduce that

$$(8) \quad 1 - \frac{\frac{\partial f}{\partial n}(k_{t+1}, n_{t+1}, z_{t+1})n_{t+1}}{y_{t+1}} = r_{t+1}^c \frac{p_t \cdot k_{t+1}}{y_{t+1}} - (1 - \delta_m) \left( \frac{p_{t+1}^m}{p_t^m} \right) \left( \frac{p_t^m k_{t+1}^m}{y_{t+1}} \right) - (1 - \delta_u) \left( \frac{p_{t+1}^u}{p_t^u} \right) \left( \frac{p_t^u k_{t+1}^u}{y_{t+1}} \right).$$

To use this relation we must use not only the return  $r_{t+1}^c$  but also the growth rate in the investment prices for the two forms of capital and the depreciation rates. From this we may produce a measure of  $p_t^u k_{t+1}^u / y_{t+1}$  using equation (8). McGrattan and Prescott (2000) use a similar method along with steady-state calculations and a model in which  $p_t^m = p_t^u = 1$  to infer the intangible capital stock.<sup>3</sup> Instead of using security market returns or historical averages of these returns, they construct physical returns, presuming that the noncorporate sector does not use intangible capital in production.<sup>4</sup> Rather than making this seemingly hard-to-defend restriction, one could link the return  $r_{t+1}^c$  directly to asset returns, as in Atkeson and Kehoe (2002). However, the practical question of which security market return to use would still be present.<sup>5</sup>

In contrast to Atkeson and Kehoe (2002) and McGrattan and Prescott (2000, 2003), uncertainty is central in the analysis of Hall (2001). For simplicity, Hall considers the case in which there is in effect a single capital stock and a single investment good, but only part of capital is measured. Equivalently, the capital stocks  $k_t^m$  and  $k_t^u$  are perfect substitutes. Thus, the production function is given by

$$y_t = f^a(k_t^a, n_t, z_t),$$

where  $k_t^a = k_t^m + k_t^u$ . Capital evolves according to

$$(9) \quad k_{t+1}^a = g^a(k_t^a, i_t^a),$$

3. McGrattan and Prescott (2000) also introduce tax distortions and a noncorporate sector. They also consider uncertainty, but with little gain. They use a minor variant of the standard stochastic growth model, and that model is known to produce physical returns with little variability.

4. McGrattan and Prescott (2003) use an a priori restriction on preferences instead of the explicit link to returns in the noncorporate sector, but this requires independent information on the preference parameters.

5. The measurement problem is made simpler by the fact that it is the composite return that needs to be computed and not the individual return on measured capital. The implied one-period returns to equity and bond holders can be combined as in Hall (2001), but computing the appropriate one-period returns for bond holders can be problematic.

with  $x_t$  excluded. The first-order conditions for investment are now given by

$$(1) \quad \frac{\partial g^a}{\partial i}(k_t^a, i_t^a) = \frac{p_t^a}{\lambda_t^a},$$

and

$$(11) \quad v_t = \frac{\lambda_t^a k_{t+1}^a}{p_t^a}$$

is measured from the security markets using the firm value relation (5) and taking investment to be numeraire. For a given  $k_t^a$ , relations (9), (10), and (11) are three equations in the three unknowns  $\lambda_t^a/p_t^a$ ,  $k_{t+1}^a$ ,  $i_t^a$ . In effect they provide a recursion that can be iterated over time with the input of firm market value  $v_t$ . Instead of returns, Hall (2001) uses asset values to deduce a time series for the aggregate capital stock and the corresponding shadow valuation of that stock.<sup>6</sup>

While Hall (2001) applies this method to estimate a time series of aggregate capital stocks, we will consider some evidence from empirical finance on return heterogeneity that indicates important differences between returns to the tangible and intangible components of the capital stocks. This suggests the consideration of models in which intangible capital differs from tangible capital in ways that might have important consequences for measurement. This includes models that are outside the adjustment cost models described here.

### 4.3 Evidence for Return Heterogeneity

We now revisit and reconstruct results from the asset-pricing literature. Since the work of Fama and French (1992) and others, average returns to portfolios formed on the basis of the ratio of book value to market value are constructed. While the book-to-market value is reminiscent of the  $q$  measure of the ratio of the market value of a firm vis-à-vis the replacement cost of its capital, here the book-to-market value is computed using only the equity holders' stake in the firm. Capital held by bond holdings is omitted from the analysis.

Recall from section 4.2 that intangible capital is reflected in only the market measure of assets but is omitted from the book measure. We are identifying firms with high intangible capital based on a high ratio of book equity to market equity (BE-ME). It is difficult to check this identification directly because the market value of debt at the firm level is not easily observed. As a check on our interpretation of the portfolios as reflecting

6. Hall (2001) establishes the stability of this mapping for some adjustment cost specifications, guaranteeing that the impact of initializing  $k_0^a$  of the recursion at some arbitrary level decays over time.

different levels of intangible capital we examined whether our portfolio construction would be different if we included debt. We used the book value of debt as an approximation to the market value and considered rankings of firms based on book assets to market assets. This resulted in essentially the same rankings of firms. In fact, the rank correlation between book assets to market assets and BE-ME averaged 0.97 over the fifty-three years of our sample. This gives us confidence in identifying the high BE-ME portfolio as containing firms with low levels of intangible and the low BE-ME portfolio as containing firms with high levels of intangibles.

Fama and French form portfolios based on the BE-ME ratio and estimate the mean return of these groups. They find that low-BE-ME groups have low average returns. Fama and French (1992) view a low BE-ME as signaling sustained high earnings and/or low risk. While we follow Fama and French (1992) in constructing portfolios ranked by BE-ME ratios, we use a coarser sort than they do. We focus on five portfolios instead of ten, but this does not change the overall nature of their findings. Each year listed firms are ranked by their BE-ME using information from COMPUSTAT. Firms are then allocated into five portfolios, and this allocation is held fixed over the following year. The weight placed on a firm in a portfolio is proportional to its market value each month.<sup>7</sup>

Firms may change groups over time, and the value weights are adjusted accordingly. In effect, the BE-ME categories are used to form five portfolio dividends, returns, and values each time period. This grouping is of course different in nature from the grouping of firms by industry standard industrial classification (SIC) codes, an approach commonly used in the industrial organization (IO) literature. For instance, firms in the low-BE-ME category may come from different industries, and the composition may change through time. On the other hand, this portfolio formation does successfully identify interesting payout heterogeneity at the firm level, as we demonstrate below.

Figure 4.1 plots the market value relative to book value of five portfolios of U.S. stocks over the period 1947–2001. Notice that there is substantial heterogeneity in the market value relative to book value of these portfolios. This potentially reflects substantial differences in intangible capital held by the firms that make up the portfolios. Further, the value of market equity to book equity fluctuates dramatically over time.

These fluctuations can reflect changes in the relative composition of the capital stock between tangible and intangible capital. They may also reflect changes in the relative valuation of the two types of capital. Changes in valuation reflect changes in conjectured productivity of the different types of capital but may also reflect changes in how the riskiness is perceived and valued by investors.

7. See Fama and French (1992) for a more complete description of portfolio construction.

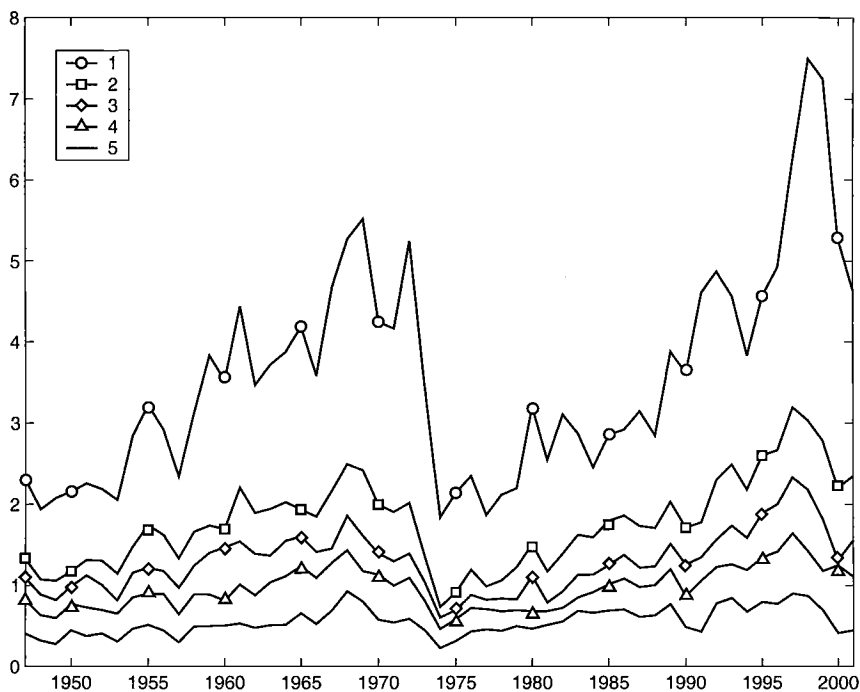


Fig. 4.1 Market-to-book value of equity for five portfolios of stocks

Table 4.1 presents sample statistics for these portfolios of stocks. For comparison, the column labeled “Market” gives statistics for the Center for Research in Securities Prices (CRSP) value-weighted portfolio. Consistent with figure 4.1, there are substantial differences in the average value of BE-ME for these portfolios. Notice that the portfolios with lower BE-ME (high market value relative to book value of equity) are also the ones with the highest level of research and development (R&D) relative to sales. This is consistent with the idea that large R&D expenditures will ultimately generate high cash flows in the future, thus justifying high current market values. Also, the high level of R&D by firms with high market valuation relative to book value may reflect substantial investment in intangibles.

While the five BE-ME portfolios are likely to have different compositions of capital, these portfolios also imply different risk-return trade-offs. As in Fama and French (1992), the low-BE-ME portfolios have lower mean returns but not substantially different volatility than high-BE-ME portfolios. The mean returns differ, and the means of implied excess returns scaled by volatility (Sharpe ratios) also differ. High-BE-ME portfolios have higher Sharpe ratios. In particular, the highest-BE-ME portfolio has a Sharpe ratio that is higher than that of the overall equity market. A

**Table 4.1** Properties of portfolios sorted by book-to-market (B-M)

	Portfolio					Market
	(1)	(2)	(3)	(4)	(5)	
Average return (%)	6.48	6.88	8.90	9.32	11.02	7.23
Standard return (%)	37.60	32.76	29.64	31.66	35.50	32.94
Average B-M	0.32	0.62	0.84	1.12	2.00	0.79
Average R&D/Sales	0.12	0.06	0.04	0.03	0.03	0.08
Sharpe ratio	0.18	0.20	0.28	0.28	0.30	0.21
Correlation with consumption	0.20	0.18	0.20	0.20	0.21	0.20

*Notes:* Portfolios formed by sorting portfolios into 5 portfolios using NYSE breakpoints from Fama and French (1992). Portfolios are ordered from lowest to highest average book-to-market value. Data from 1947Q1 to 2001Q4 for returns and B-M ratios. R&D-sales ratio is from 1950 to 2001. Returns are converted to real units using the implicit price deflator for nondurable and services consumption. Average returns and standard deviations are calculated using the natural logarithm of quarterly gross returns multiplied by four to put the results in annual units. Average book-to-market are averaged portfolio book-to-market or the period computed from COMPUSTAT. Average R&D-sales also computed from COMPUSTAT. The Sharpe ratio is based on quarterly observations. Correlation with consumption is measured as the contemporaneous correlation between log returns and log consumption growth.

portfolio with an even larger Sharpe ratio can be constructed by taking a long position in the high-BE-ME portfolio and offsetting this with a short position in the low-BE-ME portfolios. This occurs because there is substantial positive correlation across the portfolios. The spectacular Sharpe ratios that are possible have been noted by many authors; see MacKinlay (1995), for example.

The consumption-based capital asset-pricing model predicts that differences in average returns across the five portfolios are due to differences in the covariances between returns and consumption. That is, portfolios may have low returns because they offer some form of consumption insurance. The last row of table 4.1 displays the correlation between each quarterly portfolio return and the quarterly growth rate of aggregate real expenditures on nondurables and services. Because there is little difference in the volatility across portfolios, there is little difference in the implied covariance between returns and consumption growth. This measure of risk therefore implies little difference in required returns across the portfolios. The high Sharpe ratios and small covariances with consumption are known to make the consumption insurance explanation problematic; see Hansen and Jagannathan (1991) and Cochrane and Hansen (1992), for example. We will revisit this explanation, but in the context of dividend risk instead of return risk.

Differences in BE-ME are partially reflected in differences in future cash flows. Table 4.2 presents some basic properties of the dividend cash flows from the portfolios. These dividends are imputed from the CRSP return

**Table 4.2** Cash flow properties of portfolios sorted by book-to-market value

	Portfolio					Market
	(1)	(2)	(3)	(4)	(5)	
Average (log) dividend growth (%)	1.78	1.68	3.13	3.54	4.48	3.09
Standard (log) dividend growth (%)	13.50	17.09	11.71	12.05	17.76	23.99
Average log ( $D/P$ )	-3.78	-3.41	-3.23	-3.11	-3.15	n.a.
Average $P/D$	49.12	33.01	27.00	23.96	24.82	n.a.

Notes: n.a. = not available.  $D/P$  = dividend-price ratio.  $P/D$  = price-dividend ratio.

files. Each month and for each stock, CRSP reports a return without dividends, denoted  $R_{t+1}^{wo} \equiv P_{t+1}/P_t$ , and a total return that includes dividends, denoted  $R_{t+1}^w \equiv (P_{t+1} + D_{t+1})/P_t$ . The dividend yield  $D_{t+1}/P_t$  is then imputed as

$$\frac{D_{t+1}}{P_t} = R_{t+1}^w - R_{t+1}^{wo}.$$

Changes in this yield along with the capital gain in the portfolio are used to impute the growth in portfolio dividends. This construction has the interpretation of following an initial investment of \$1 in the portfolio and extracting the dividends while reinvesting the capital gains. From the monthly dividend series we compute quarterly averages. Real dividends are constructed by normalizing nominal dividends on a quarterly basis by the implicit price deflator for nondurable and service consumption taken from the national income and product accounts. Finally, some adjustment must be made to quarterly dividends because of the pronounced seasonal patterns in corporate dividend payout. Our measure of quarterly dividends is constructed by taking an average of the logarithm of dividends in a particular quarter and over the previous three quarters. We average the logarithm of dividends because our empirical modelling will be linear in logs. Table 4.2 reports statistics for this constructed proxy of log dividends.

Notice from table 4.2 that the low-BE-ME portfolios also have low dividend growth. Just as there is considerable heterogeneity in the measures of average returns, there is also considerable heterogeneity in growth rates. An important measurement question that we will explore is whether these *ex post* sample differences in dividend growth are something that is fully perceived *ex ante* or whether some of this heterogeneity is the outcome of dividend processes with low-frequency components. We suspect that much of the observed heterogeneity in dividend growth was known a priori by investors and hence this heterogeneity will reflect potential differences in risk. Some of our calculations that follow will treat this heterogeneity as reflecting in part differences in long-run risk. In section 4.4, we turn to a discussion of risk measurement for these cash flows.

A potential concern in evaluating dividends at the portfolio level is that portfolio formation could lead to artificial differences in the long-run risk properties of portfolio cash flows that are not easily interpretable. For example, it may appear that portfolios biased toward investing in stocks with low dividend growth will necessarily have low growth rates in cash flows and therefore little long-run exposure to economic growth. Notice, however, that the implied dividend growth rates in the constructed portfolios depend in part on the relative prices of stocks bought and sold as the composition of the portfolios change over time. Stocks with temporarily low dividend growth rates will have relatively high price appreciation, which can offset the low growth rates. Thus, the portfolio formation might actually result in a more stable dividend or cash flow.

#### 4.4 Dividend Risk

In asset pricing it is common to explore risk premia by characterizing how returns covary with a benchmark return, as in the capital asset pricing model (CAPM), or, more generally, how returns covary with a candidate stochastic discount factor. The focus of the resulting empirical investigations are on return risk, in contrast to dividend or cash-flow risk.

Recently there has been an interest in understanding cash-flow risk using linear time series methods. Examples include the work of Bansal, Dittmar, and Lundblad (2002a,b) and Cohen, Polk, and Vuoteenaho (2002). We follow this literature by using linear time series methods to motivate and construct a measure of dividend risk.

To use linear time series methods requires a log-approximation for present discounted value formulas as developed by Campbell and Shiller (1988a,b).<sup>8</sup> Write the one-period return in an equity as

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\left(\frac{1 + P_{t+1}}{D_{t+1}}\right) \frac{D_{t+1}}{D_t}}{\frac{P_t}{D_t}},$$

where  $P_t$  is the price and  $D_t$  is the dividend. Take logarithms and write

$$r_{t+1} = \log\left(\frac{1 + P_{t+1}}{D_{t+1}}\right) + (d_{t+1} - d_t) - (p_t - d_t),$$

where lowercase letters denote the corresponding logarithms. Next, approximate

8. Santos and Veronesi (2001) suggest a model for studying cash flow risk that avoids linear approximation by instead adopting a nonlinear model of income shares.

$$\log\left(\frac{1 + P_{t+1}}{D_{t+1}}\right) \approx \log[1 + \exp(\overline{p-d})] + \frac{1}{1 + \exp(\overline{d-p})}(p_{t+1} - d_{t+1} - \overline{p-d}),$$

where  $\overline{p-d}$  is the average logarithm of the price dividend ratio. Use this approximation to write

$$(12) \quad r_{t+1} - (d_{t+1} - d_t) = \chi + \rho(p_{t+1} - d_{t+1}) - (p_t - d_t),$$

where

$$\rho \equiv \frac{1}{1 + \exp(\overline{d-p})}.$$

As shown by Campbell and Shiller (1988a), this approximation is reasonably accurate in practice.

Treat equation (12) as a difference equation in the log price dividend ratio and solve this equation forward:

$$p_t - d_t = \sum_{j=0}^{\infty} \rho^j (d_{t+1+j} - d_{t+j} - r_{t+1+j}) + \frac{\chi}{1 - \rho}.$$

This relation says that a time  $t + 1$  shock to current and future dividends must be offset by the same shock to returns in the sense of a present discounted value. The discount factor  $\rho$  will differ depending on the average logarithm of the dividend-price ratio for the security or portfolio. The present discounted value restriction is mathematically the same as that developed by Hansen, Roberds, and Sargent (1991) in their examination of the implications of present-value budget balance.

To understand this restriction, posit a moving-average representation for the dividend growth process and the return process:

$$\begin{aligned} d_t - d_{t-1} &= \eta(L)w_t + \mu_d \\ r_t &= \kappa(L)w_t + \mu_r. \end{aligned}$$

Here  $\{w_t\}$  is a vector, independently and identically distributed (i.i.d.) standard normal process and

$$\eta(z) = \sum_{j=0}^{\infty} \eta_j z^j, \quad \kappa(z) = \sum_{j=0}^{\infty} \kappa_j z^j,$$

where  $\eta_j$  and  $\kappa_j$  are row vectors.

Since  $p_t - d_t$  depends only on date- $t$  information, future shocks must be present-value neutral:

$$(13) \quad \kappa(\rho) - \eta(\rho) = 0.$$

For instance, if returns are close to being i.i.d., but not dividends, then

$$(14) \quad \kappa(0) \approx \eta(\rho).$$



The discounted dividend response should equal the return response to a shock. Since in fact returns are predictable, we will present some evidence that bears on this approximation.

To evaluate the riskiness of each portfolio's exposure to the shocks  $w_t$ , we also measure the impact of the shocks on consumption growth:

$$c_t - c_{t-1} = \gamma(L)w_t + \mu_c,$$

where  $c_t$  is the logarithm of aggregate consumption. To measure the economic magnitude of return responses, Hansen and Singleton (1983) used the familiar representative agent model with constant relative risk aversion (CRRA) utility:

$$(15) \quad E \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\theta} R_{t+1}^j \mid F_t \right] = 1,$$

where  $\theta$  is the coefficient of relative risk aversion. Under a lognormal approximation, the return on portfolio  $j$  satisfies

$$r_{t+1}^j = \kappa^j(L)w_{t+1} + \mu_r^j.$$

Euler equation (15) then implies that  $\mu_r^j$  satisfies

$$(16) \quad E[r_{t+1}^j \mid F_t] - r_t^f = -\frac{\kappa^j(0) \cdot \kappa^j(0)}{2} + \theta\gamma(0) \cdot \kappa^j(0).$$

where  $r_t^f$  is the logarithm of the risk-free return.

Whereas Hansen and Singleton (1983) used equation (16) to study directly one-period return risk, Bansal, Dittmar, and Lundblad (2002a,b) instead looked at the discounted dividend risk. In most of this paper we follow Bansal, Dittmar, and Lundblad (2002a) and treat  $\eta^j(\rho)$  as a measure of risk in dividend growth. We refer to this measure as *discounted dividend risk*. The present-value relation (abstracting from approximation error) implies that this combination of dividend responses to future shocks must be offset by the corresponding return responses. As  $\rho$  tends to 1, we refer to the limit  $\eta(1)$  as long-run risk.

We will not include returns in our vector autoregressions for the reasons explained by Hansen, Roberds, and Sargent (1991).<sup>9</sup> We will sometimes include dividend-price ratios in the vector autoregressive systems, however. These ratios are known to be informative about future dividends. Write implied moving-average representation as

$$p_t^j - d_t^j = \xi^j(L)w_t + \mu_p^j.$$

We may then back out a return process (approximately) as

9. Hansen, Roberds, and Sargent (1991) show that when returns are included in a VAR, the restrictions in equation (13) cannot be satisfied for the shocks identified by the VAR unless the VAR system is stochastically singular.

$$r_t^j = \kappa^j(L)w_t + \mu_r^j,$$

where

$$\kappa^j(z) = (\rho - z)\xi^j(z) + \eta^j(z).$$

It follows from this formula for  $\kappa^j$  that the present-value-budget balance restriction in equation (13) is satisfied by construction and is not testable.

To summarize, we use  $\eta^j(\rho)$  as our measure of discounted dividend risk. When dividend-price ratios are also included in the VAR system, the present-value budget balance restriction in equation (13) is automatically satisfied. By construction, discounted return risk and discounted dividend risk coincide.

#### 4.5 Measuring Dividend Risk Empirically

In this section we evaluate the riskiness of the five BE-ME portfolios using the framework of section 4.4. Riskiness is measured by the sensitivity of portfolio cash flows and prices to different assumptions made to identify aggregate shocks. Since we are interested in the long-run impact of aggregate shocks, we consider several VAR specifications that make different assumptions about the long-run relationships between consumption, portfolio cash flows, and prices. In particular, we examine the effects of moving from the assumption of little long-run relationship between aggregates and portfolio cash flows to the assumption that there is a cointegrated relationship between aggregate consumption and cash flows.

##### 4.5.1 Empirical Model of Consumption and Dividends

To measure dividend risk we require estimates of  $\gamma$  and  $\eta$ . We describe how to obtain these using VAR methods for consumption and dividends. The least restrictive specification we consider is

$$\mathbf{A}_0 x_t + \mathbf{A}_1 x_{t-1} + \mathbf{A}_2 x_{t-2} + \dots + \mathbf{A}_\ell x_{t-\ell} + \mathbf{B}_0 = w_t,$$

where consumption is the first entry of  $x_t$  and the dividend level is the second entry. The vectors  $\mathbf{B}_0$  and  $\mathbf{B}_1$  are two-dimensional, and similarly the square matrices  $\mathbf{A}_j, j = 1, 2, \dots, \ell$  are two by two. The shock vector  $w_t$  has mean zero and covariance matrix  $\mathbf{I}$ . We normalize  $\mathbf{A}_0$  to be lower triangular with positive entries on the diagonals. Form:

$$\mathbf{A}(z) \doteq \mathbf{A}_0 + \mathbf{A}_1 z + \mathbf{A}_2 z^2 + \dots + \mathbf{A}_\ell z^\ell.$$

We are interested in specification in which  $\mathbf{A}(z)$  is nonsingular for  $|z| < 1$ .

We identify the first shock as the consumption innovation, and our aim is to measure the discounted average response:

$$\eta(\rho) = (1 - \rho) \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{A}(\rho)^{-1}.$$

We use these formulas to produce long-run risk measures for each back-to-market portfolio.

We also compute the limiting responses as  $\rho$  tends to unity. While we want to allow for  $A(z)$  to be singular at unity, we presume that  $(1 - z)A(z)^{-1}$  has a convergent power series for a region containing  $|z| \leq 1$ . This is equivalent to assuming that both consumption and dividends are (at least asymptotically) stationary in differences. The limiting responses are thus contained in the matrix

$$(1 - z)A(z)^{-1} \Big|_{z=1}.$$

When  $A(1)$  is nonsingular, the limiting response matrix is identically zero, but it will be nonzero when  $A(1)$  is singular. The matrix  $A(1)$  is nonsingular when the VAR does not have stochastic growth components. When it is singular, the vector time series will be *cointegrated* in the sense of Engle and Granger (1987). We will explore singular specifications of  $A(1)$  in which difference between log consumption and log dividends is presumed to be stationary.

#### 4.5.2 Data Construction

For our measure of aggregate consumption we use aggregate consumption of nondurables and services taken from the national income and product accounts (NIPA). This measure is quarterly from 1947Q1 to 2002Q4, is in real terms, and is seasonally adjusted. Portfolio dividends were constructed as discussed in section 4.3. For portfolio prices in each quarter we use end-of-quarter prices.

Motivated by the work of Lettau and Ludvigson (2001) and Santos and Veronesi (2001), in several of our specifications we allow for a second source of aggregate risk that captures aggregate exposure to stock market cash flows. This is measured as the share of corporate cash flows in aggregate consumption and is measured as the ratio of corporate earnings to aggregate consumption. Corporate earnings are taken from NIPA.

In all of the specifications that follow, the VAR models were fit using five quarters of lags.<sup>10</sup> See appendix A for more details of the data construction.

#### 4.5.3 Bivariate Model of Consumption and Dividends

First we follow Bansal, Dittmar, and Lundblad (2002b) and consider bivariate regressions that include aggregate consumption and the dividends for each portfolio separately. Table 4.3 reports results for the case where the state variable  $x_t$  is given by<sup>11</sup>

10. We also conducted some runs with nine lags. With the exception of the results for portfolio 1 when using aggregate earnings, the results were not greatly effected.

11. Notice that we consider separate specifications of the state variable for each portfolio. Ideally, estimation with all of the portfolio cash flows would be interesting, but because of data limitations this is not possible.

**Table 4.3** Discounted responses of portfolio dividends in a log-level VAR

	Portfolio				
	(1)	(2)	(3)	(4)	(5)
Discount factor	0.9943	0.9918	0.9902	0.9889	0.9894
<i>A: Consumption shock</i>					
OLS estimator	0.14	0.34	0.22	0.53	1.32
10 percentile	-0.05	0.06	0.01	0.38	0.98
30 percentile	0.07	0.20	0.15	0.47	1.16
Median	0.14	0.30	0.22	0.53	1.33
70 percentile	0.21	0.43	0.30	0.61	1.53
90 percentile	0.34	0.72	0.41	0.75	1.92
<i>B: Dividend shock</i>					
OLS estimator	0.68	1.23	0.75	0.58	1.19
10 percentile	0.45	0.66	0.56	0.42	0.78
30 percentile	0.56	0.87	0.65	0.50	0.98
Median	0.67	1.09	0.74	0.58	1.18
70 percentile	0.85	1.45	0.86	0.69	1.48
90 percentile	1.32	2.23	1.12	0.96	2.29

$$(17) \quad x_t = \begin{bmatrix} c_t \\ d_t \end{bmatrix}.$$

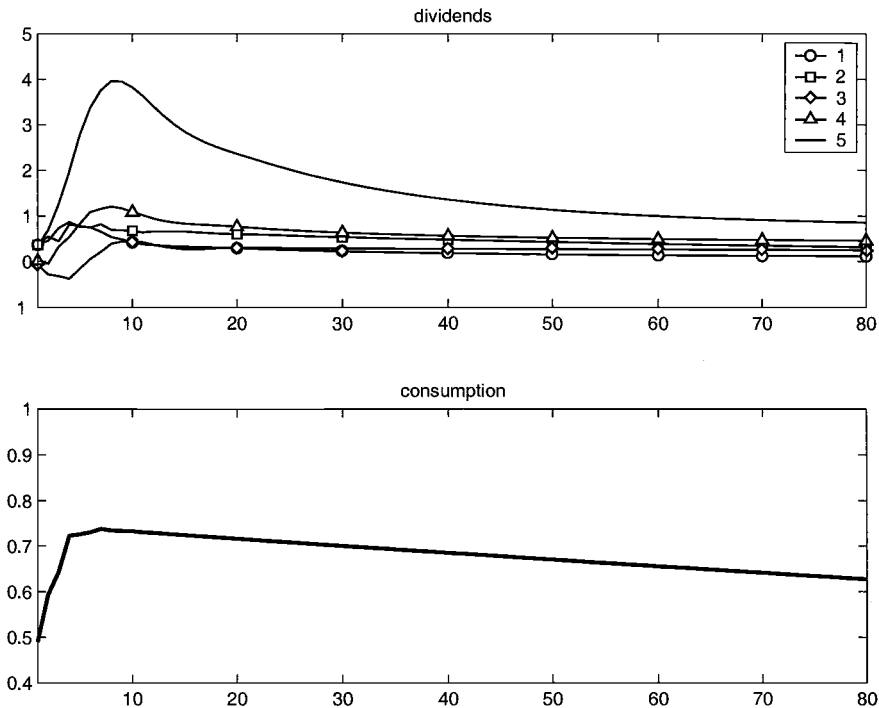
For notational convenience we do not display the dependence of  $x_t$  and hence  $\mathbf{A}(z)$  on the choice of portfolio.

To simplify the interpretation of the shock vector  $w_t$ , we initially restrict the matrix  $\mathbf{A}(z)$  to be lower triangular. Under this restriction, consumption depends only on the first shock while dividends depend on both shocks. This recursive structure presumes that consumption is not “caused” by dividends in the sense of Granger (1969) and Sims (1972).<sup>12</sup>

The first row of panel A shows that according to the discounted measure of dividend risk, the high book-to-market returns have a larger measure of dividend vis-à-vis the low book-to-market returns. The differences are quite striking in that the response to a consumption shock increases almost ten times in comparing portfolio 1 and portfolio 5. This ordering was noted by Bansal, Dittmar, and Lundblad (2002a), using a different set of restrictions on the VAR.<sup>13</sup> To illustrate the portfolio differences more fully, consider figure 4.2. This figure displays the implied responses of log dividends to a consumption shock. The discounted measure of risk reported in table

12. When this restriction on  $A(z)$  is relaxed, the measured discounted responses that we report below to a consumption shock are essentially the same.

13. Bansal, Dittmar, and Lundblad (2002a) consider two types of regressions. In the first, dividend growth is regressed on an eight-quarter moving average of past consumption growth. In the second, detrended dividends are regressed on contemporaneous detrended consumption and four leads and lags of consumption growth.



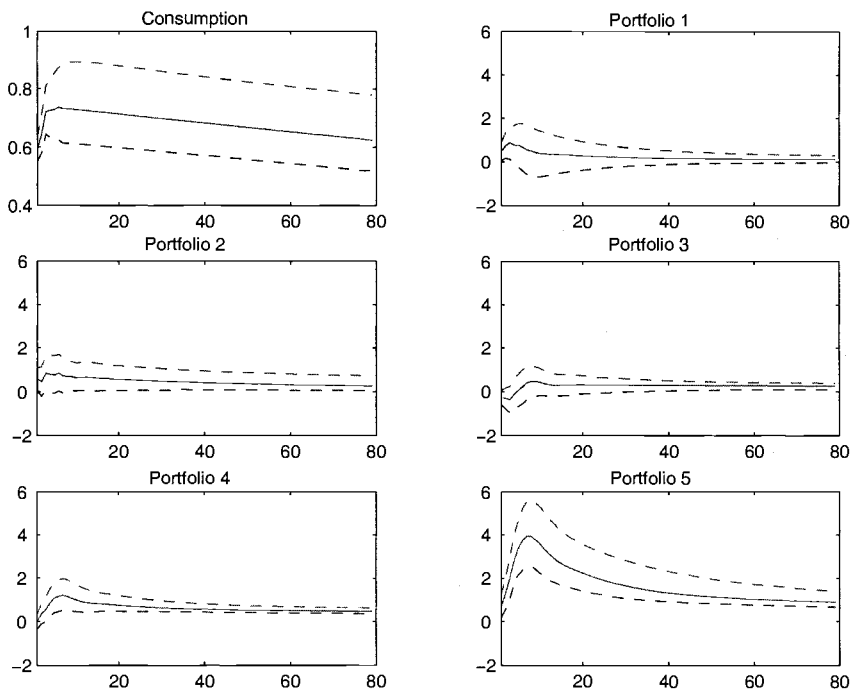
**Fig. 4.2** Impulse responses to a consumption shock

*Notes:* This figure reports the impulse response functions for each of the portfolios and for consumption obtained by estimating the log-level version of the VAR in which  $x_t$  has entries  $c_t$  and  $d_t$ . The matrix  $A(z)$  is restricted to lower triangular.

4.3 is a weighted average of the responses depicted in figure 4.2. Notice in particular that portfolio 5 has a substantially different response to a consumption shock with a pronounced peak response at about the ten-quarter horizon. The half-lives of the discount factors range between sixteen years for portfolio 4 to thirty years for portfolio 1. As a result, the discounted average responses weight heavily tail responses.

Table 4.3 also reports Bayesian posterior percentiles for the discounted consumption risk computed using the method described in appendix B. These percentiles provide a measure of accuracy. Figure 4.3 plots the 10th, 50th, and 90th percentile for the individual impulse responses. Notice that these measures of accuracy imply substantial sampling error in the estimated discounted responses. For example, consider the results for portfolios 1 and 5 as displayed in figure 4.3. Although the estimated short-run response to a consumption shock is quite different across these two portfolios, the confidence intervals narrow this difference substantially.

Next we explore singular specifications of  $A(1)$  in which the difference between log consumption and log dividends is presumed to be stationary.



**Fig. 4.3 Bayesian percentile for impulse responses to a shock to consumption**

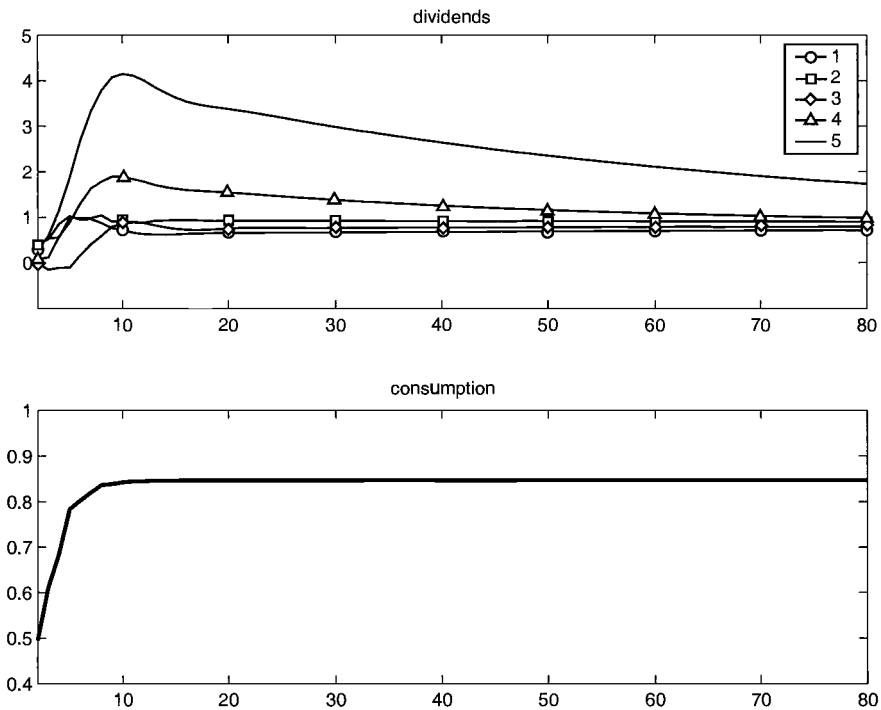
*Notes:* This figure gives the 10th, 50th, and 90th percent percentile for the impulse response function depicted in figure 4.2. The upper left panel depicts the consumption response, and the other five panels depict the responses for each of the five portfolio cash flows.

Again we use VAR methods, but now the first variable is the first-difference of log consumption and the second is the difference between log consumption and log dividends. This specification is in effect a restriction on  $A(z)$  and in particular restricts  $A(1)$  to be rank one. We continue to assume that  $A(z)$  is lower triangular. Since the ratio of dividends to consumption is presumed to be stationary for all portfolios, the long-run response of dividends to a consumption shock is the same. The discounted responses can still differ, however.

As table 4.4 demonstrates, when dividends and consumption are restricted to respond the same way to permanent shocks, the discounted risk measures increase relative to those computed without restricting the rank of  $A(z)$ . The limiting response is about .82 for all portfolios. The discounted responses of portfolios 1, 2, and 3 to a consumption shock are all pulled toward this value. The discounted risk measures for portfolios 4 and 5 are also increased by imposing this limiting value on the impulse response. In figure 4.4 we depict the impulse responses when cointegration is imposed and consumption is restricted. Comparing the impulse responses to a consumption shock in this figure to those in figure 4.2, we see that while tail

**Table 4.4** Discounted responses of portfolio dividends in a cointegrated VAR

	Portfolio				
	(1)	(2)	(3)	(4)	(5)
Discount factor	0.9943	0.9918	0.9902	0.9889	0.9894
<i>A: Consumption and permanent shock</i>					
OLS estimator	0.75	0.89	0.75	1.11	2.03
10 percentile	0.13	0.40	0.27	0.80	1.24
30 percentile	0.59	0.71	0.60	0.97	1.62
Median	0.82	0.91	0.80	1.12	1.97
70 percentile	1.04	1.11	0.99	1.28	2.43
90 percentile	1.39	1.44	1.31	1.61	3.41
<i>B: Dividend and transitory shock</i>					
OLS estimator	2.60	2.06	2.18	1.14	2.43
10 percentile	1.31	1.40	1.24	0.72	1.49
30 percentile	1.72	1.68	1.59	0.91	1.90
Median	2.19	1.96	1.92	1.10	2.30
70 percentile	2.86	2.30	2.36	1.40	2.89
90 percentile	4.15	2.91	3.13	2.09	4.21



**Fig. 4.4** Impulse responses to a consumption shock for the cointegrated specification

*Notes:* The impulse responses are identified by a VAR estimated with  $c_t - c_{t-1}$  and  $c_t - d_t$  as the components of  $x_t$ . The matrix  $A(z)$  is restricted to be lower triangular.

**Table 4.5** Discounted responses of portfolio dividends to permanent and transitory shocks

	Portfolio				
	(1)	(2)	(3)	(4)	(5)
Discount factor	0.9943	0.9918	0.9902	0.9889	0.9894
<i>A: Discounted responses, permanent shock</i>					
OLS estimator	3.62	2.86	2.76	1.50	-0.14
10 percentile	1.61	1.62	1.16	0.45	-1.41
30 percentile	2.36	2.20	0.60	0.98	-0.47
Median	3.08	2.67	0.80	1.35	-0.05
70 percentile	4.07	3.22	0.99	1.89	0.30
90 percentile	5.88	4.07	1.31	3.09	0.86
<i>B: Discounted responses, transitory shock</i>					
OLS estimator	-0.63	-0.56	-0.66	-1.01	1.68
10 percentile	-1.36	-1.26	-1.49	-1.45	1.13
30 percentile	-0.92	-0.87	-0.97	-1.03	1.34
Median	-0.68	-0.62	-0.70	-0.83	1.52
70 percentile	-0.45	-0.33	-0.42	-0.64	1.76
90 percentile	0.02	0.15	0.08	-0.38	2.25
<i>C: Covariance between consumption and discounted response</i>					
OLS estimator	0.29	0.36	0.32	0.58	0.63

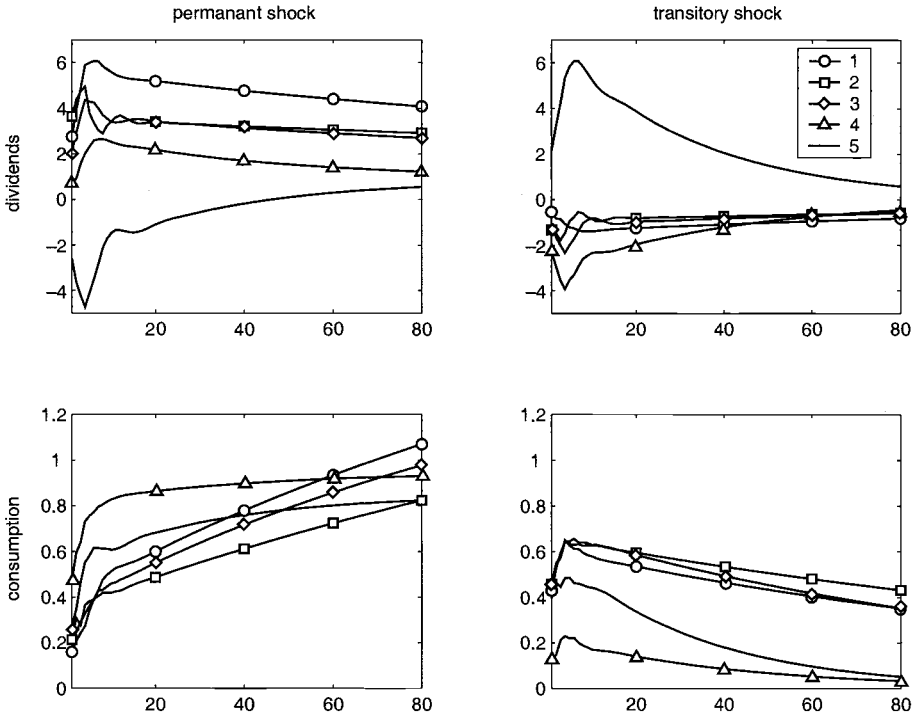
properties of the impulse responses have been altered, portfolio 5 continues to have a peak response at about ten quarters.<sup>14</sup>

For the cointegrated systems, we consider an alternative identification scheme. We do not restrict  $A(z)$  to be lower triangular, but we instead identify a permanent and transitory shock following an approach suggested by Blanchard and Quah (1989). In this case the permanent shock is the shock that has the same nonzero long-run effect on consumption and dividends. The second shock is chosen to be orthogonal to the permanent shock and has zero long-run impact on consumption and dividends by construction. We also normalize the shocks so that positive movements in both shocks induce positive movements in consumption. Since both shocks influence consumption, both shocks are pertinent in assessing the riskiness of the implied cash flows.

The results are reported in table 4.5, and the impulse responses are depicted in figure 4.5. The discounted responses to the permanent consumption shock differ from the response to the previously identified consumption shocks. For instance, portfolio 5 now has an initial negative response to permanent consumption shock, and this persists for many periods. The discounted response remains negative for this portfolio even though the

14. Given the data transformation, the Bayesian posterior percentile for the VAR are based on a different specification of the prior coefficient distribution over comparable coefficients.





**Fig. 4.5 Impulse responses to a permanent and transitory shock**

*Notes:* The impulse responses are identified by a VAR estimated with  $c_t - c_{t-1}$  and  $c_t - d_t$  as the two components of  $x_t$ . The permanent shock is identified as the shock that alters consumption permanently but is exactly offset in the long run by movement dividends. The transitory shock is uncorrelated with the permanent shock and has a transitory impact on both consumption and dividends. A line with circles is used to depict the portfolio 1 response, a line with squares for portfolio 2, a line with diamonds for portfolio 3, a line with triangles for portfolio 4, and a solid line for portfolio 5.

limiting response is by construction positive. Thus, holding portfolio 5 appears to provide some insurance against consumption risk, which makes the large mean return appear puzzling. The other portfolios' dividends respond positively to this consumption shock. The portfolio 5 response to transitory shock is always positive, however. This is in contrast to the other four portfolios, which have negative responses to this shock. The transitory shock contributes to the discounted riskiness of the dividends.

A defect of this identification scheme is that the identified shocks differ depending upon which portfolio we use in the empirical investigation. To address this concern the final panel of table 4.5 reports values of the term  $\gamma(0)' \eta(\rho)$  for each portfolio. This gives the conditional covariance between consumption growth and the discounted dividends. This accumulation of the effects of the two shocks mirrors our previous results. Risk increases

from portfolio 1 to portfolio 5, although the largest increase is from portfolio 3 to 4 and then from portfolio 4 to 5.

The discounted dividend risk measures suggest that the high book-to-market portfolios have more longer-run covariation with consumption as measured by discounted responses. As emphasized by Bansal, Dittmar, and Lundblad (2002a), this provides a qualitative explanation for the heterogeneity in mean returns. The discounted dividend riskiness of the high book-to-market returns must be compensated for by a higher mean return. This claim is *qualitative* for at least two reasons. First, if returns are predictable, then the conditional means of returns will not equal the unconditional means reported in table 4.1. Second, while the discounted dividend response is approximately equal to discounted response of cumulative returns, if returns are predictable then the discounted return response will differ from the one-period return response that is pertinent for asset pricing. In the next section we report some evidence on return predictability.

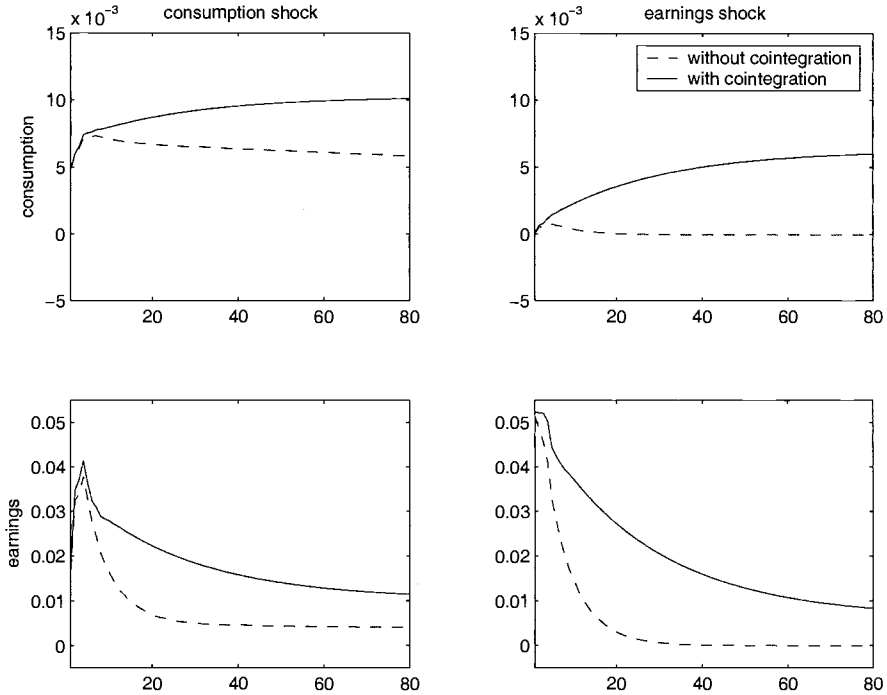
#### 4.6 Results with an Additional Aggregate Shock

In our final VAR specification we consider what happens when an additional aggregate shock is added. In this specification  $x_t$  is given by

$$x_t = \begin{bmatrix} c_t \\ e_t \\ p_t \\ d_t \end{bmatrix},$$

where  $e_t$  is corporate profits at time  $t$ . We are led to consider this latter variable by the empirical investigations of Lettau and Ludvigson (2001) and Santos and Veronesi (2001). These authors argue for the addition of an aggregate share variable to help account for asset values. For example, Santos and Veronesi argue that exposure to stock market risk is affected by the contribution of corporate payouts to aggregate consumption. We restrict  $A(0)$  to be a lower-triangular matrix with positive entries on the diagonal. We refer to the first shock as the consumption shock and the second one as the earnings shock. Following Santos and Veronesi, we also consider a model in which the earnings-consumption ratio is stationary.<sup>15</sup> In this specification we restrict the upper left two-by-two block of  $A(1)$  to be singular by running a VAR using the first difference of log consumption and the contemporary difference between earnings and consumption as data.

15. Santos and Veronesi (2001) do not use linear VAR model but rather pose a share model in which the counterpart to the earnings-consumption ratio is restricted to be between zero and one.



**Fig. 4.6 Impulse responses of consumption and earnings to a consumption shock and an earnings shock**

*Notes:* The impulse responses without imposing cointegration were constructed from a bivariate VAR with entries  $c_t, e_t$ . These responses are given by the dashed lines (----). Solid lines (—) are used to depict the impulse responses estimated from a cointegrated system. The impulse response functions are computed from a VAR with  $c_t - c_{t-1}$  and  $e_t - d_t$  as time series components.

To avoid parameter proliferation, we restrict the dynamics of the aggregate variables  $c_t$  and  $e_t$  to not be Granger-caused by the individual portfolio dividends and prices. That is, we restrict  $A(z)$  to be block lower triangular. We consider the discounted responses to a shock to consumption and a shock to earnings.

Figure 4.6 displays how consumption and earnings respond to the respective shocks in the two models. The dashed lines are for the cases where there is no restriction on the relative growth of earnings and consumption (“without cointegration”). The solid lines are for the cases where any permanent shock has the same long-run impact on consumption and earnings (“with cointegration”). Under our identification of shocks, the shock to earnings has no immediate effect on consumption. In section 4.4 we assumed that preferences over consumption are separable over time. The resulting pricing relationship, equation (16), predicts that any exposure of

cash flows to the earnings shock will have no impact on average security returns.

Figure 4.6 shows that there is a substantial change in the long-run variation of consumption when we move to the model with cointegration. Both shocks have persistent effects on consumption and hence earnings (through the cointegration restriction). Moreover, the earnings shock now has a quantitatively important impact on consumption. More general models of preferences predict that the effect of a shock on future consumption will have a significant effect on risk exposure. One such example is the recursive utility specification considered, for example, by Epstein and Zin (1989). In this model the effect of a shock on future consumption affects attitudes to risk through the influence of the shock on the continuation value of utility. For this reason the shock to aggregate earnings could be a very important shock in accounting for the riskiness of cash flows.

We now explore the implied responses to the financial variables. The discounted responses of portfolio dividends and prices are reported in table 4.6. The discounted dividend responses to a consumption shock in the model without cointegration are very similar to those we reported in table 4.3. This is to be expected because the earnings shock has little impact on consumption for this system. In the model with cointegration, the response to a consumption shock increases for each portfolio, but the ordering across portfolios is not clear. For example, portfolio 3 now has the largest response. Further, notice that with cointegration the discounted responses to an earnings shock are much larger than without cointegration. In general, the mixed results of table 4.6 indicate substantial sensitivity in measures of dividend risk both to the specification of the long-run dynamics of the series and to the definition of shocks.

Panel A of table 4.6 does provide some evidence that differences in the discounted response to a consumption shock could provide an explanation for differences in the observed average returns of the portfolios. We now examine the more ambitious quantitative question of how much risk aversion is required for this explanation to work. Using equation (16) we can compute an implied value of  $\theta$  from the difference in the risk premia between any two portfolios. We use the discounted dividend responses as estimates of  $\kappa^j(0)$  based on approximation (14). Recall that this presumes that there is little predictability in returns, about which we will have more to say subsequently. In what follows we use the risk premium of each portfolio relative to that of portfolio 5 to calculate  $\theta$ , ignoring the restriction that the same value of  $\theta$  should be used in explaining the entire cross-section of average returns. Our goal is merely to provide a convenient metric to evaluate the quantitative significance of observed differences in discounted dividend responses.

Results are reported in table 4.7 based on the risk measures of panel A of table 4.6. The discounted dividend responses do imply higher returns for

**Table 4.6** Discounted responses in four variate VAR with aggregate profits

	Portfolio				
	(1)	(2)	(3)	(4)	(5)
Discount factor	0.9943	0.9918	0.9902	0.9889	0.9894
<i>A: Dividends to consumption shock (without cointegration)</i>					
OLS estimator	0.05	0.17	0.42	0.57	1.11
10 percentile	-0.34	-0.08	0.25	0.38	0.75
30 percentile	-0.04	0.09	0.34	0.48	0.95
Median	0.07	0.18	0.41	0.57	1.12
70 percentile	0.16	0.26	0.50	0.68	1.33
90 percentile	0.29	0.42	0.64	0.90	1.81
<i>B: Dividends to earnings shock (without cointegration)</i>					
OLS estimator	0.02	0.08	0.32	0.00	0.04
10 percentile	-0.43	-0.26	0.10	-0.26	-0.47
30 percentile	-0.09	-0.03	0.23	-0.11	-0.17
Median	0.05	0.09	0.33	0.00	0.05
70 percentile	0.15	0.19	0.44	0.12	0.31
90 percentile	0.30	0.37	0.64	0.38	0.85
<i>C: Dividends to consumption shock (with cointegration)</i>					
OLS estimator	0.64	1.20	1.56	1.47	1.37
10 percentile	-0.05	0.32	1.00	0.98	0.54
30 percentile	0.70	0.92	1.27	1.23	0.92
Median	1.04	1.26	1.53	1.46	1.25
70 percentile	1.39	1.66	1.93	1.75	1.72
90 percentile	2.22	2.55	2.80	2.44	2.88
<i>D: Dividends to earnings shock (with cointegration)</i>					
OLS estimator	0.46	1.22	1.99	1.18	0.32
10 percentile	-0.62	-0.09	1.17	0.43	-0.91
30 percentile	0.52	0.82	1.57	0.81	-0.30
Median	1.00	1.33	1.97	1.14	0.16
70 percentile	1.52	1.94	2.55	1.59	0.81
90 percentile	2.80	3.32	4.05	2.64	2.49

**Table 4.7** Implied value of risk aversion coefficient  $\theta$ 

	Portfolio			
	(1)	(2)	(3)	(4)
$\theta$	222	228	159	163

Note: These values of  $\theta$  are computed using the discounted risk measures reported in panel A of table 4.6.

**Table 4.8** Return responses in four variate VAR with aggregate profits

	Portfolio				
	(1)	(2)	(3)	(4)	(5)
Discount factor	0.9943	0.9918	0.9902	0.9889	0.9894
	<i>A: No cointegration</i>				
On impact	2.15	1.89	1.75	1.77	2.24
Discounted	0.05	0.17	0.42	0.57	1.11
	<i>B: Cointegration</i>				
On impact	1.98	1.93	1.78	1.75	2.14
Discounted	0.64	1.20	1.56	1.47	1.37

the higher book-to-market portfolios, but the magnitude of the risks is small. As a result, we calculate high implied coefficients of relative risk aversion. This result has been noted extensively in the finance literature.

By including prices along with dividends in the VAR, we have ensured that the present-value budget balance restriction is satisfied by construction. Any shock to log dividends must be offset by a shock to log returns through discounting provided log returns are measured via the log approximation of section 4.4. While we have focused our attention on how dividends respond to economically meaningful shocks, we now consider returns. In table 4.8 we compare the discounted return response (equal to the discounted dividend response from panel A of table 4.6) to the on-impact return response to a consumption shock. It is this latter response that is pertinent for the consumption-based asset pricing model described previously. As previously mentioned, in models based on recursive utility, the intertemporal composition of risk is known to matter. While the discounted return response increases across the portfolios, the same is not true of the on-impact return response. As in Hansen and Singleton (1983), the on-impact return response to consumption does not help to explain the observed heterogeneity in a cross section of portfolio returns.<sup>16</sup> The discounted response is still potentially of interest in models where there is some type of delay in the consumption response due to adjustment costs or some type of misspecification of the model (see, e.g., Daniel and Marshall 1997).

As the results of table 4.8 indicate, there is potentially important predictability in returns. This predictability has led the finance literature to decompose variation in price-dividend ratios into components based on predicted returns and predicted dividend growth (see, e.g., Campbell 1991 and Cochrane 1991a). We apply this decomposition to the book-to-market portfolios.

16. Hansen and Singleton (1983) based their findings on industry portfolios and not the book-to-market portfolios studied here.

As before, instead of using actual returns, we use the implied returns from the log-linear approximation. In this case, the present-value formula,

$$p_t - d_t = \sum_{j=0}^{\infty} \rho^j (d_{t+1+j} - d_{t+j} - r_{t+1+j}) + \frac{\chi}{1 - \rho},$$

holds by construction provided the growth in the estimated VAR is dominated by the discount factor  $\rho$ . Of course, this same relation holds once expectations are taken:

$$p_t - d_t = \sum_{j=0}^{\infty} \rho^j E(d_{t+1+j} - d_{t+j} | F_t) - \sum_{j=0}^{\infty} \rho^j E(r_{t+1+j} | F_t) + \frac{\chi}{1 - \rho}.$$

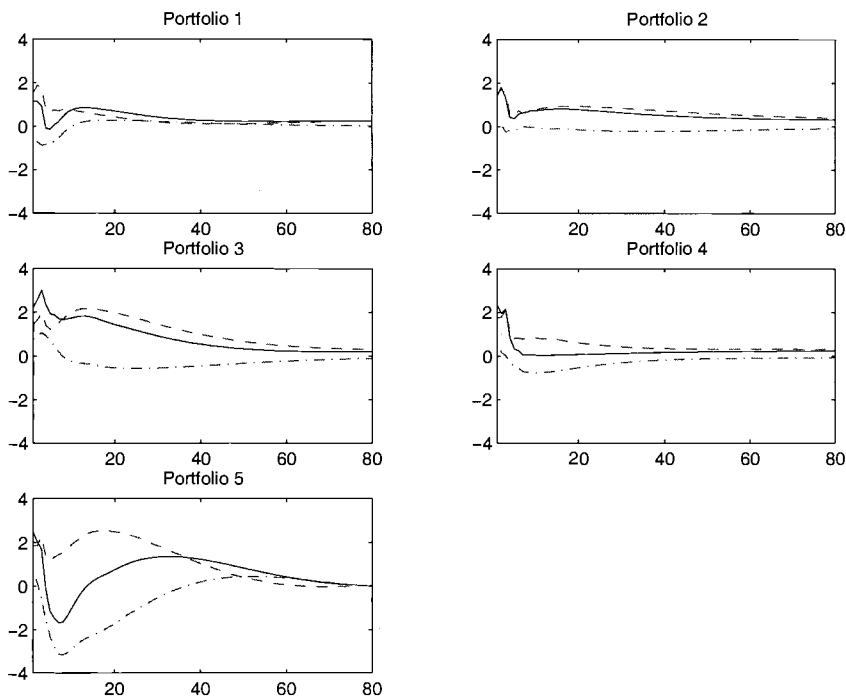
The right-hand side gives us an ad hoc decomposition of the price-dividend ratio in terms of predicted future dividends and predicted future returns. We use this decomposition to account for the impulse-response functions of the price-dividend ratio. Any response of the price-dividend ratio to a shock must be due to the differential response of discounted expected dividend growth and discounted expected returns to that same shock.<sup>17</sup> In figure 4.7 we plot the impulse response functions for the portfolio price-dividend ratios for a consumption shock for each of the five portfolios. Neither return nor dividend predictability dominates explanations of price-dividend variation across portfolios. While the return contribution is much more pronounced for portfolio 2, the dividend contribution is particularly important for portfolio 5.

#### 4.7 Conclusions

In this paper we reviewed two findings pertinent for using asset market data to make inferences about the intangible capital stock. We presented evidence familiar from the empirical finance literature that returns are heterogeneous when firms are grouped according to their ratio of market equity to book equity. This evidence suggests that there are important differences in the riskiness of investment in measured capital vis-à-vis intangible capital. This has potentially important ramifications for how to build explicit economic models to use in constructing measurements of the intangible capital stock.

A risk-based interpretation of return heterogeneity requires more than just a model with heterogeneous capital. It also requires a justification for the implied risk premiums. There has been much interest recently in the finance literature on using VAR methods to understand riskiness of serially correlated cash flows or dividends. The discounted dividend risk-measures using VAR methods find that high book-to-market portfolio returns have more economically relevant risk. The discounted responses are larger for

17. See appendix C for a more complete discussion of this calculation.



**Fig. 4.7** Decomposition of price-dividend responses to a consumption shock

*Notes:* These estimates were constructed from a VAR with entries  $c_t$ ,  $e_t$ ,  $p_t$ , and  $d_t$ . The matrix  $A(z)$  is restricted to be lower triangular in blocks of two, and  $A(0)$  is lower triangular. For each portfolio, a solid line (—) depicts the impulse response of  $p - d$ , a dashed line (-----) depicts the impulse response of the dividend component of  $p - d$ , and a dashed and dotted line (---) depicts the impulse response of return component of  $p - d$ .

these portfolios. Moreover, the dividend response to a consumption shock for portfolio 5, a portfolio of the highest book-to-market returns, stands out relative to other dividend responses. The impulse response for this portfolio has a pronounced peak at around ten quarters. This peak is present in many of the VAR specifications of shocks. The shock responses are very different when we identify a permanent-transitory decomposition of shocks to consumption and dividends, but portfolio 5 still looks different relative to the other portfolios. Further, our results are sensitive to other specifications of the long-run dynamics and to our identification of the shocks.

The empirical evidence we report follows the finance literature by focusing on the claims of equity holders. As emphasized by Hall (2001), what is pertinent for measurement purposes is the combined claims of bond holders and equity holders. It is the overall value of the firm or enterprise that



is pertinent. Similarly, this analysis focuses on dividends as the underlying claims of equity holders and not on overall cash flows of the firms. The risk associated with broader-based cash flow measures are of considerable interest for future research.

## Appendix A

### *Data Appendix*

#### Consumption

We use aggregate consumption of nondurables and services taken from NIPA<sup>18</sup> (table 2.2). The quarterly data are seasonally adjusted at annual rates, deflated by the implicit price deflator for nondurable and services consumption.

#### Corporate Earnings

Corporate earnings are measured as corporate profits with inventory valuation and capital consumption adjustments from NIPA (table 1.14), the quarterly data are deflated by the implicit price deflator for nondurable and services consumption.

#### BE-ME Portfolios<sup>19</sup>

We follow Fama and French (1992) in constructing portfolios ranked by book-to-market ratios. Five BE-ME portfolios are formed at the end of each June using New York Stock Exchange (NYSE) breakpoints. The BE used in June of year  $t$  is the book equity for the last fiscal year end in  $t - 1$ . ME is price times shares outstanding at the end of December of  $t - 1$ . We use all NYSE, American Stock Exchange (AMEX), and Nasdaq stocks for which we have ME for December of  $t - 1$  and June of  $t$ , and BE for  $t - 1$ . For each stock, monthly returns with and without dividends and monthly market values are taken from the CRSP monthly stock data set. We take annual BE data from 1950 to 2001 from the CRSP/COMPUSTAT merged industrial data set.<sup>20</sup> We thank Kenneth French for providing us with the annual BE data from 1926 to 1950. The two data sets are merged together with the CRSP data set using CRSP Permanent Company Number

18. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

19. SAS codes used to construct the portfolio monthly returns, BE-ME, and R&D-sale are available upon request.

20. CRSP monthly stock data set and CRSP/COMPUSTAT merged data set are from Wharton Research Data Services, University of Pennsylvania.

(PERMNO). For each portfolio, the monthly returns with and without dividends from July of year  $t$  to June of year  $t - 1$  are weighted averages of the stock returns with and without dividends in the same period, using the ME in June of year  $t$  as the weights. Portfolio book-to-market ratios in year  $t$  are the weighted averages of the stock book-to-market ratios in year  $t$ . R&D and sales data are taken from COMPUSTAT, available from 1950 to 2001. In year  $t$ , portfolio R&D-sales ratio is the weighted average of the R&D-Sales of each firm in year  $t$ .

### Dividends<sup>21</sup>

The dividends' yield  $D_{t+1}/P_t$  is imputed from portfolio returns with dividend  $R_{t+1}^w \equiv (P_{t+1} + D_{t+1})/P_t$  and returns without dividend  $R_{t+1}^{wo} \equiv (P_{t+1} + D_{t+1})/P_t$  as following

$$\frac{D_{t+1}}{P_t} = R_{t+1}^w - R_{t+1}^{wo}.$$

Change in this yield along with the capital gain in the portfolio is used to impute the growth in portfolio dividends

$$\frac{D_{t+1}}{D_t} = \frac{\frac{D_{t+1}}{P_t}}{\frac{D_t}{P_{t-1}}} \cdot \frac{P_t}{P_{t-1}} = \frac{R_{t+1}^w - R_{t+1}^{wo}}{R_t^w - R_t^{wo}} R_t^{wo}.$$

From the dividend growth we impute the dividend level except for the initial value,

$$\frac{D_{t+1}}{P_0} = \prod_{s=1}^t \frac{D_{s+1}}{D_s} \cdot \frac{D_1}{P_0}.$$

From monthly dividend series we compute the quarterly average. We initialize the dividends in 1947Q1 such that the dividend for the market portfolio in 1947Q1 is same as the corporate earning in 1947Q1, and the BE-ME portfolio dividends are proportional to the market portfolio with respect to the market value. We then take twelve months' trailing average because of the pronounced seasonal pattern in the corporate dividend payout. Our measure of quarterly dividends in quarter  $t$  is constructed by taking an average of the logarithm of dividends in quarter  $t$  and over the previous three quarters  $t - 3$ ,  $t - 2$ ,  $t - 1$ . We average the logarithm of dividends instead of levels because our empirical modelling will be linear in logs. This construction has the interpretation of following an initial investment of \$1 in the portfolio and extracting the dividends while reinvesting the capital gains.

21. Portfolio dividends, prices, and returns series used in this paper are available upon request.

Returns and dividends are converted to real units using the implicit price deflator for nondurable and services consumption.

### Price Deflator

The nominal consumption, corporate earning, portfolio returns, and portfolio dividends are deflated by the implicit price deflator for nondurable and services consumption, which is the weighted average of the personal nondurable consumption implicit price deflator  $P_t^{CN}$  (1996 = 100) and personal services consumption implicit price deflator  $P_t^{CS}$  (1996 = 100), taken from NIPA table 7.1. The weights are determined by the relative importance of nominal nondurable consumption ( $CN_t$ ) and service consumption ( $CS_t$ ); that is,

$$P_t^C = \frac{P_t^{CN}CN_t + P_t^{CS}CS_t}{CN_t + CS_t}.$$

## Appendix B

### *Bayesian Confidence Intervals*

Consider the VAR

$$A(L)y_t + C_0 = w_t,$$

where  $y_t$  is  $d$ -dimensional. The matrix  $A(0)$  is lower triangular. We base inferences on systems that can be estimated equation by equation. The  $w_t$  is a normal random vector with mean zero and covariance matrix  $I$ . We follow Sims and Zha (1999) and Zha (1999) by considering a uniform prior on the coefficients. Given the recursive nature of our model, we may follow Zha (1999) by building the joint posterior for all parameters across all equations as a corresponding product. This requires that we include the appropriate contemporary variables on the right-hand side of the equation to ensure that  $w_{t+1}$  has the identity as the covariance matrix. In effect we have divided the coefficients of the VAR into blocks that have independent posteriors given the data. We construct posterior confidence intervals for the objects that interest us as nonlinear functions of the VAR coefficients.<sup>22</sup>

We computed the posterior confidence intervals using Monte Carlo methods using characterizations in Zha (1999) and Box and Tiao (1973). Confidence intervals are centered around the posterior median computed in our simulation; the error bands are computed using the 10th and 90th

22. In making the prior uniform over all coefficients, we follow a suggestion by but not the actual practice in Sims and Zha (1999). There is a minor difference evident in the discussion in Sims and Zha; see page 1142.

**Table 4B.1** Unstable fraction of simulation (%)

	Portfolio				
	(1)	(2)	(3)	(4)	(5)
Log-level VAR	2	14	1	1	3
Cointegrated VAR	22	11	21	6	10
Cointegrated unrestricted VAR	20	14	19	8	2
Four variate VAR	11	9	2	7	5
Four variate cointegrated VAR	38	24	5	13	19

percentile. Our numbers are based on 100,000 simulations, taking out the unstable systems. The unstable fractions of the simulated systems for different models we used are reported in table 4B.1.

## Appendix C

### *Decomposition of Price-Dividend Ratios*

We decompose the dividend price ratios into two components. Recall that

$$p_t - d_t = \xi(L)w_t + \mu_p,$$

where

$$(z - \rho)\xi(z) = \eta(z) - \kappa(z),$$

and

$$\kappa(\rho) = \eta(\rho).$$

Write

$$\xi(z) = \frac{\eta(z) - \eta(\rho)}{z - \rho} - \frac{\kappa(z) - \kappa(\rho)}{z - \rho}.$$

The functions

$$\frac{\eta(z) - \eta(\rho)}{z - \rho}, \quad \frac{\kappa(z) - \kappa(\rho)}{z - \rho}$$

have one-sided power series convergent for  $|z| < 1$ . In particular, we interpret

$$-\frac{\kappa(z) - \kappa(\rho)}{z - \rho}$$

as the transform of the moving-average coefficients for the return contribution to the price-dividend ratio, and

$$\frac{\eta(z) - \eta(\rho)}{z - \rho}$$

as the dividend contribution to the price-dividend ratio. The coefficients of these transforms give us a corresponding additive decomposition of the impulse response function for the logarithm of the price-dividend ratio.

Consider a VAR in which the logarithm of the price-dividend ratio and the logarithm of dividend growth are included. It is simplest to work with the VAR written as a first-order system:

$$\begin{aligned} X_{t+1} &= \mathbb{A}X_t + \mathbb{C}w_{t+1} \\ d_t &= \mathbb{H}_d X_t \\ p_t &= \mathbb{H}_p X_t \end{aligned}$$

with an expanded state vector. The impulse response function for the price-dividend ratio to the first shock is given by

$$(\mathbb{H}_p - \mathbb{H}_d)\mathbb{A}^j\mathbb{C}e_1,$$

where  $e_1$  is a vector of zeros except in the first position, where there is a unit coefficient. The impulse response function for the expected discounted dividend growth is given by

$$\mathbb{H}_d[(\mathbb{A} - I)(I - \rho\mathbb{A})^{-1}]\mathbb{A}^j\mathbb{C}e_1.$$

The impulse response function for the return contribution is

$$[\mathbb{H}_d(1 - \rho)\mathbb{A}(I - \rho\mathbb{A})^{-1} - \mathbb{H}_p]\mathbb{A}^j\mathbb{C}e_1.$$

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## Comment      Susanto Basu

The paper by Hansen, Heaton, and Li (henceforth HHL) contains a number of important concepts and interesting results, but the pieces never quite come together to form a coherent picture. At the end of the day, we are left somewhat puzzled as to exactly what the authors were trying to do and what they believe they have actually done. These comments are my attempt to supply those missing connections.

HHL wish to contribute to the literature on “intangible capital,” presented recently in its most provocative form by Hall (2001). My interpretation of this paper is that the authors agree that Hall's approach is potentially valid, but then point out—and test—additional asset-pricing implications of his model in an attempt to assess its plausibility.

Hall begins from the observation that in the steady state of a neoclassical model with endogenous capital accumulation, one can infer the quan-

tity of a firm's capital from its value in financial markets. When there are positive but reasonable costs of adjusting the capital stock, one needs to make a modest adjustment, because the value of a firm will exceed the value of its capital if investment is positive. Thus, as HHL review in their paper, Hall can use the static first-order condition for investment from the model of Hayashi (1982) to infer the total quantity of capital, both tangible and intangible.

But the static equation for investment is not the only first-order condition of Hayashi's model; there is also a dynamic first-order condition, the Euler equation. In a stochastic, discrete-time formulation (unlike Hayashi's deterministic, continuous-time model), that Euler equation can be written as

$$(1) \quad E_t \left[ \frac{F_K(K_{t+1}, L_{t+1}) - \delta q_{t+1}}{q_t} + \frac{q_{t+1} - q_t}{q_t} \right] = 1 + r_{t+1} + \theta_{t+1}.$$

In this equation,  $E_t$  is the expectation of future variables conditional on time- $t$  information;  $F$  is the firm's production function;  $q$  is the marginal value of a unit of installed capital;  $r$  is a risk-free real interest rate; and  $\theta$  is a risk premium appropriate for discounting the stochastic cash flow of the firm (where  $\theta$  may be negative in the case of a "negative-beta" asset).

Hall (2001) needs to assume Hayashi's (1982) conditions to measure marginal  $q$  from asset prices, since in general asset prices measure only Tobin's average  $q$ . But if one can measure marginal  $q$  from asset prices, then one can also estimate equation (1) using asset market data. For simplicity, suppose a firm that is 100 percent equity financed. Then the left-hand side of equation (1) is the expected return to holding equity—expected dividends plus expected capital gains, divided by the purchase price of capital to create a percentage return. Given a model that relates  $\theta$  to observables (for example, the consumption CAPM), one can test whether the predictions of equation (1) hold. Thus, in a sense HHL are pointing out an over-identifying restriction of Hall's intangible-capital model, which can be tested with asset market data.

The major uncertainty regarding Hall's paper has always been whether the asset values of firms in the late 1990s reflected large amounts of intangible capital—with the capital gains of those years interpreted as equally large investments in intangibles—or whether the high stock values and large capital gains were the classic symptoms of an unsustainable bubble ("irrational exuberance") in equity markets. In principle, one can answer this question using aggregate time series data to construct the holding returns in equation (1). If the holding returns, over long periods of time, are in line with the historical norms for equity returns, then Hall's rational model of valuation will be validated. If, on the other hand, the 1990s are followed by a long period of abnormally low returns, then the evidence will favor the mispricing model.



The problem with this strategy, of course, is that one would have to wait for at least ten years to conduct a test with enough data to settle this issue convincingly. Since such a delay is completely unacceptable for policy purposes, HHL propose an interesting alternative. They note that the defining characteristic of equity values in the late 1990s was a low ratio of book-to-market values of stocks. (Hall, of course, interpreted this observation as indicating a high ratio of intangible to tangible capital, since only the latter are recorded on firms' books.) HHL note further that one can observe similarly low book-market ratios in the historical cross section of equities. Thus, assuming that the features that led to a low book-market (B-M) ratio for the aggregate stock market in the 1990s are the same forces that lead some firms to have low B-M ratios in the cross-section distribution—a big assumption, but a reasonable hypothesis to investigate—we can use known facts about the subsequent returns of firms with low B-M values to shed some light on the intangible-capital versus mispricing debate.

On a first look, the cross-sectional evidence strengthens the case for mispricing. Shleifer (2000, chap. 5) reviews a number of empirical papers in this area. All find that value stocks (ones with high B-M ratios) outperform growth or glamour stocks (those with low B-M ratios). However, HHL investigate a new possibility suggested by several recent empirical finance papers. These papers argue that stocks with low B-M ratios are actually less risky, and thus their lower expected returns can be justified by risk-based considerations. That is, in terms of equation (1), HHL note that a low ex post return is justified by a lower risk premium,  $\theta$ . As the authors note, in the intangible-capital context this story requires that intangible capital be less risky than physical capital.

However, my reading of the paper is that HHL's attempt to investigate this hypothesis turns up little solid evidence in favor of the risk-based explanation for the low returns of growth stocks. The results are quite fragile, and there seem to be few findings that are robust to reasonable alternative empirical specifications. Since these recent papers on risk-based explanations for the low returns of growth stocks have excited considerable interest in the finance literature, this lack-of-robustness result alone makes HHL's paper quite valuable.

Furthermore, it is important to realize that the risk-based story requires that investors consciously accept lower expected returns ex ante, in exchange for the supposed reduction in risk. Thus, according to the risk-based story, surveys of investors in the late 1990s should show that investors expected lower returns going forward.<sup>1</sup>

1. The dinner-table conversation implied by the risk-based model is something like the following: "Honey, today I put all our retirement assets in new-economy, Nasdaq stocks. I know, I know—we would do much better investing in old-economy companies. But, by golly, this is safe as a house!" The spouse's reaction is left to the imagination of the reader.

However, the evidence strongly contradicts this hypothesis. Vissing-Jorgensen (2004) analyzes surveys of future returns that consumers expected, starting in 1998. Figure 1 in her paper contradicts the low-expected-return hypothesis at every point. Expected one-year returns were incredibly optimistic in 2000 and fell sharply in 2001–2. Of course, B-M ratios were high in 2000 and lower in 2001–2, so the relation of expected returns to B-M ratios is exactly the opposite of what is needed to rationalize the risk-based explanation. In fact, the evidence in Vissing-Jorgensen’s paper is quite consistent with a model where investors “chase returns” by naively extrapolating recent high returns into the future. This evidence does not bode well for Hall’s intangible-capital interpretation of the high stock values in the 1990s.

In sum, the paper by Hansen, Heaton, and Li is a valuable contribution to empirical finance, because it shows that the recent challenge to a long-standing finance puzzle, the value stock premium, is itself subject to challenge. The econometric evidence in this paper is a good complement to the survey-based evidence I have just discussed. Together, the two suggest that the torrid stock market of the late 1990s reflected a classic bubble and not rational valuation of large quantities of intangible capital. But only time will tell for certain.

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