Uncertainty Prices

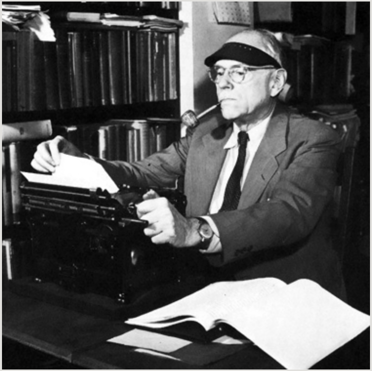
When Beliefs Are Tenuous

Lars Peter Hansen, *University of Chicago*

Thomas J. Sargent, *New York University*

ISB Hyderabad Finance Conference
July 29, 2016

Frank Knight and Uncertainty (1921)



“We must infer what the future situation would be without our interference, and what changes will be wrought by our actions. Fortunately, or unfortunately, *none* of these processes is *infallible*, or indeed ever *accurate* and *complete*.”

Placing Uncertain Investors Inside an Economic Model

When *constructing* a dynamic economic model, researchers:

- depict economic actors (consumers, enterprises) as they cope with uncertainty when making economic decisions with future consequences
- deduce the resulting market responses and consequences for resource allocations

Rational Expectations Inside an Economic Model

Muth (1961) and Lucas (1972): Economic actors (investors) use *long histories* of data to infer the model, including its parameters.

- ▷ Yields a stochastic notion of *equilibrium* with expectations determined *inside the model*
- ▷ Gives a coherent approach to *policy analysis*

Influential, but *neglects* some components of uncertainty by featuring only *risk*. Statistical challenges are off the table.

Risk Inside the Model

- Recent empirical successes rely on endowing investors with knowledge of statistically subtle components of the macro time series. Where does this confidence come from?
- Imposes stochastic volatility exogenously.
- Imposes large risk aversion.

Success?

Uncertainty in the Macroeconomy

- IMF Report, July 2016:

  > *The Brexit vote implies a substantial increase in economic, political, and institutional uncertainty, which is projected to have negative macroeconomic consequences, especially in advanced European economies.*

- Obstfeld (IMF chief economist), July 2016:

  > *The real effects of Brexit will play out gradually over time, adding elements of economic and political uncertainty that could be resolved only after many months. This overlay of extra uncertainty, in turn, may open the door to an amplified response of financial markets to negative shocks.*

Secular Stagnation?

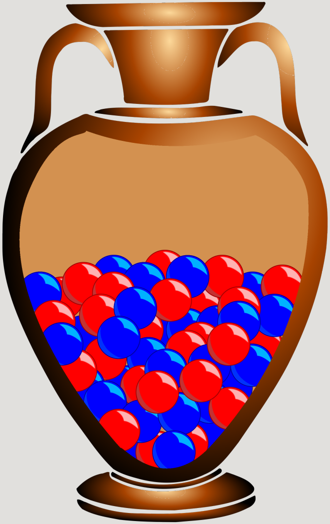
Joel Mokyr

"There are a myriad of reasons why the future should bring more technological progress than ever before – perhaps the most important being that technological innovation itself creates questions and problems that need to be fixed through further technological progress." (2013)

Robert Gordon

"…the rise and fall of growth are inevitable when we recognize that progress occurs more rapidly in some time periods than others…The 1870-1970 century was unique: Many of these inventions could only happen once, and others reached natural limits." (2016)

Uncertainty Can Be *Risk*



50 Red Balls

50 Blue Balls

Uncertainty Can Be *Ambiguity*

? Red Balls

? Blue Balls

Uncertainty Can *Change Over Time*

? Red Balls

? Blue Balls

Uncertainty Can Be Complex

Las Meninas, Diego Velàzquez

Statistical Complexity

- When is it challenging to learn and draw inferences?
- When is there more scope for behavioral distortions?
- When could statistical uncertainty induce fluctuations in prices of uncertainty that are observed in financial markets?

Take a broader perspective than is typical in economic analyses.

Multiple Components to Uncertainty

- Model *risk* - what probabilities does a model assign to events in the future?

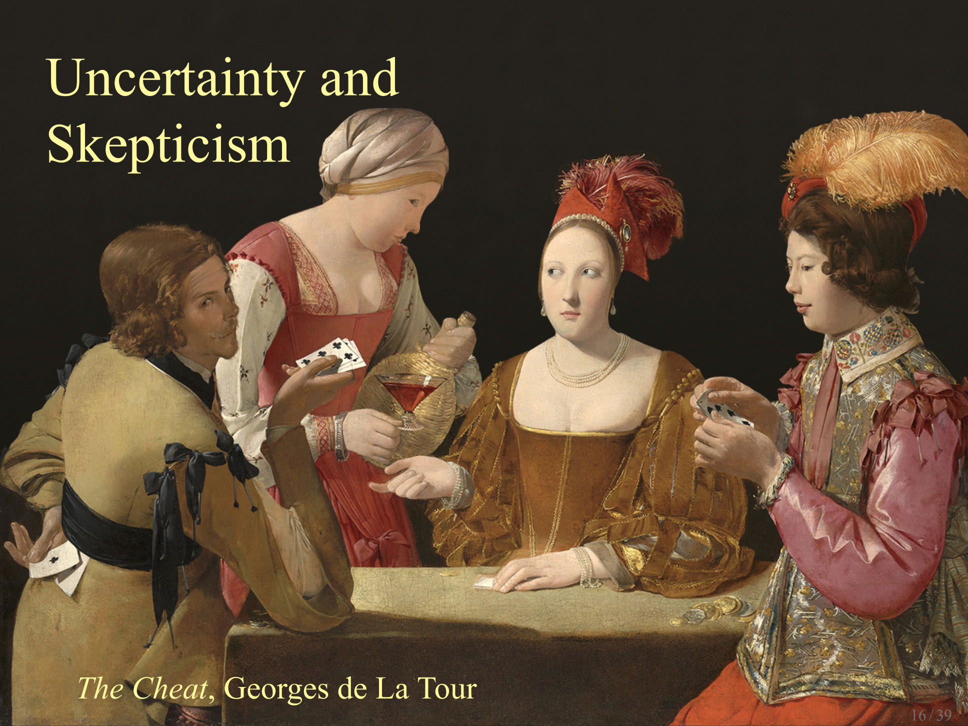
Multiple Components to Uncertainty

- Model *risk* - what probabilities does a model assign to events in the future?
- Model *ambiguity* - how much confidence do we place in each model?

Multiple Components to Uncertainty

- Model *risk* - what probabilities does a model assign to events in the future?
- Model *ambiguity* - how much confidence do we place in each model?
- Model *misspecification* - how do we use models that are not perfect?

Uncertainty and Skepticism



The Cheat, Georges de La Tour

Evidence from Financial Market Data

Private sector observation: Risk-On Risk-Off

- Investors' appetites for risk rise and fall over time

Academic research: Time-varying expected returns

- Measured risk-return tradeoffs from financial markets fluctuate over time
- "Risk-prices" are bigger in magnitude some times than others

This evidence poses a challenge to model builders: what explains these movements?

Using Decision Theory to Broaden the Notion of Uncertainty

- Investors struggle with how to perceive the future in a meaningful way
- They approach this struggle with differing degrees of confidence in their beliefs
- The impact of the struggle varies over time as new evidence or perspectives emerge

Outcome: New sources for fluctuations in *uncertainty prices* emerge in models of financial markets. Concerns about long-term uncertainty influence even short-term pricing.

Overview of the Implementation

Aim:

- Allow for multiple benchmark models with parameters that change over time.
- Surround a family of benchmark models with a set that includes statistically similar models
- Study how concerns about misspecification alter market prices of uncertainty

Application

Ingredients:

- Family of benchmark models for log consumption growth with a predictable growth state variable
- Set of alternative less structured models
- Tractable robust decision problems for planner and representative investor

Outcome:

- Endogenous source of uncertainty price variation
- Endogenous nonlinearity in valuation

Cast of Characters

- Baseline models and nonparametric alternatives
- Positive martingales that represent alternative probabilities
- Penalize deviations from the baseline models using statistical discrimination

References

- Anderson, Hansen, and Sargent, 2003, "A Quartet of Semigroups for Model Specification, Robustness, Prices of Risk, and Model Detection." *JEEA*.
- Chen and Epstein, 2002, "Ambiguity, Risk, and Asset Returns in Continuous Time." *Econometrica*.
- Gilboa and Schmeidler, 1989, "Maxmin Expected Utility with Non-unique Prior." *Journal of Mathematical Economics*.
- Maccheroni, Marinacci and Rustchini, 2006, "Dynamic Variational Preferences." *Journal of Economic Theory*.
- Petersen, James, and Dupuis, 2000, "Minimax Optimal Control of Stochastic Uncertain Systems with Relative Entropy Constraints," *Automatic Control, IEEE Trans. Aut. Control*
- Newman and Stuck, 1979, "Chernoff Bounds for Discriminating between Two Markov Processes," *Stochastics*.

Alternative Probabilities

Use H to represent alternative probabilities via positive martingales

- Baseline probabilities - W is a Brownian motion
- Martingale:

$$M_t^H = \exp\left(\int_0^t H_u \cdot dW_u - \frac{1}{2}\int_0^t H_u \cdot H_u du\right)$$

- Evolution: $dM_t^H = M_t^H H_t \cdot dW_t$.
- Implied perturbed probabilities: $E^H\left[B_t|\mathcal{F}_0\right] = E\left[M_t^H B_t|\mathcal{F}_0\right]$
- Implied perturbed evolution of W:

$$dW_t = H_t dt + dW_t^H$$

where dW_t^H is a standard Brownian increment under the H probability measure

Normal shocks dW_t with history dependent distortions H_t to the drift

Sets of Benchmark Models

Use $H = R$ to denote a benchmark probability and impose restrictions on R. Resulting family of martingales is $\mathcal{M}^o$.

Two sources of time variation

- Robust Bayesian explores the prior sensitivity. Worst case priors change over time depending on the perspective of the decision maker
- Underlying parameters change making learning difficult if not impossible

Dynamic Consistency

Good (1952):

> *In what circumstances is a minimax solution reasonable? I suggest that it is reasonable if and only if the least favorable initial distribution is reasonable according to your body of beliefs.*

- Explore prior sensitivity and changing parameters via instant by instant constraints on R_t's.

  Note: May not be able to produce an implied worst-case model as is typically featured in robust Bayesian analysis for time invariant parameter specifications
- Allow for model misspecification using a penalization approach

Example

- Initial model parameterized by $\hat{\mu}, \hat{\phi}, \hat{\beta}, \hat{\kappa}, \sigma_y, \sigma_z$

$$dY_t = (.01)\left(\hat{\alpha}_y + \hat{\beta} Z_t\right) dt + (.01)\sigma_x \cdot dW_t$$

$$dZ_t = \hat{\alpha}_z dt - \hat{\kappa} Z_t dt + \sigma_z \cdot dW_t$$

- *W* a Brownian motion
- Think of *Y* as log consumption and use logarithmic utility
- *Z* generates "long-run risk" or growth rate uncertainty

A Family of Parametric Alternative Models

- Let $H = R$ index the drift distortion of alternative restricted models
- Parametric alternatives

$$dY_t = 0.01\left(\alpha_y + \beta Z_t\right)dt + 0.01\sigma_y \cdot dW_t^R$$
$$dZ_t = \alpha_z dt - \kappa Z_t dt + \sigma_z \cdot dW_t^R$$

- Construct M^R where $R_t = \eta(Z_t) \equiv \eta_0 + \eta_1 Z_t$
  where

$$\sigma = \begin{bmatrix} (\sigma_y)' \\ (\sigma_z)' \end{bmatrix}$$

  and

$$\sigma\eta_0 = \begin{bmatrix} \alpha_y - \hat{\alpha}_y \\ \alpha_z - \hat{\alpha}_z \end{bmatrix}$$
$$\sigma\eta_1 = \begin{bmatrix} \beta - \hat{\beta} \\ \hat{\kappa} - \kappa \end{bmatrix}$$

Alternative Probabilities

- Benchmark models are parametric alternatives including time-varying specifications. Represent using martingales parameterized by R's and restricted via:

$$R_t \in \Xi_t.$$

  where Ξ_t is chosen so as to include statistically similar parametric models.
- Represent other statistically similar probabilities parameterized by H's.

Equilibrium Construction

- Solve a "robust planners problem"
- Deduce restrained worst case model
- Compute shadow prices including the price of uncertainty

Outcome: worst-case drift distortion $H_t^* = \eta^*(X_t)$.

$$\text{"local risk price"} \quad = \quad \underbrace{.01\sigma_y}_{\text{risk price}} \quad + \quad \underbrace{(-H_t^*)}_{\text{uncertainty price}}$$

In contrast to the first term, the second term fluctuates over time.

Set of Reference Models

Construction of Ξ_t

- Use relative entropy to construct sets of parametric models
- Allow parameters to vary within these sets

Robust Planner's Problem

$x = (y, z)$ and m is a martingale realization. $\hat{\mu}(x)$ is the composite drift, U measures utility and δ captures the subjective discount rate and $\mathcal{D}(x)$ restricts the choice of the benchmark models.

- Hamilton-Jacobi-Bellman equation

$$
\begin{aligned}
0 = \min_{h, r \in \mathcal{D}(x)} & -\delta m V(x) + mU(x) + m\hat{\mu}(x)' \frac{\partial V}{\partial x}(x) \\
& + m h' \sigma' \frac{\partial V}{\partial x}(x) \\
& + \frac{m}{2} \text{trace} \left[\sigma' \frac{\partial^2 V}{\partial x \partial x'}(x) \sigma \right] + \frac{\theta m}{2} |h - r|^2 .
\end{aligned}
$$

  $\sigma h + \hat{\mu}$ is an alternative drift, $\sigma r + \hat{\mu}$ is a drift for a reference model, and $mV(x)$ is the value function.

- Compute V after scaling by $\frac{1}{m}$.

Statistical Calibration

Hansen-Heaton-Li VAR style model matched to quarterly consumption data and data on business related income and personal dividends projected onto our parametric class

- project onto our parametric class by matching long-term implications
- correlation between the shock to consumption and the shock to the growth rate in consumption $\sigma_y \cdot \sigma_z \neq 0$

Income Relative to Consumption

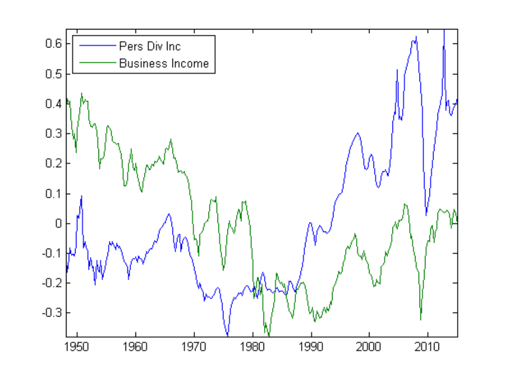


Figure: Business income is measured as corporate profits plus proprietor income. Personal dividend income is the aggregate dividends paid to individuals.

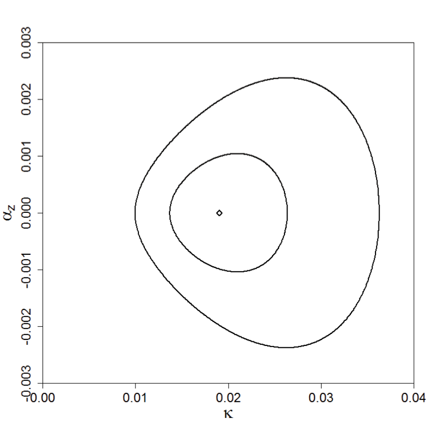
Families of Benchmark Models

We consider two alternative model selection problems under ambiguity aversion based on two alternative families of reference models.

- Alternative (α_z, κ)'s subject to a relative entropy constraint. Worst case model depends on the state z but not on the value function. (Shown in what follows)
- Alternative (β, κ)'s subject to a relative entropy constraint. Worst case model depends on the state z and on the value function.

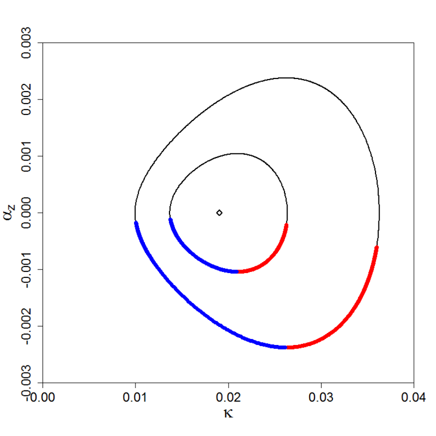
Outcome: Worst case models are Markovian with nonlinear dynamics. High persistence in the growth state is feared in bad times (negative values of z) and low persistence in good times (positive values of z).

Parameter Contours for (α_z, κ)



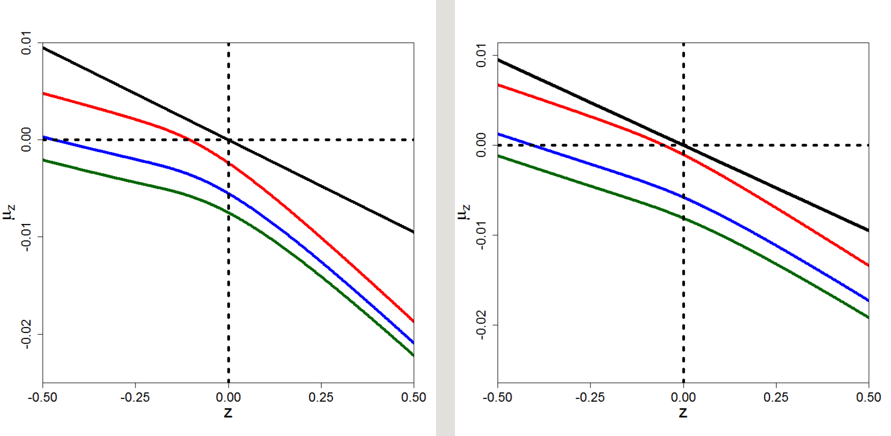
The outer curve $\sqrt{2 \times \text{rel entropy}}$ is 0.0707 and for the inner curve it is 0.0354. The small diamond in the model depicts the approximating model.

Parameter Contours for (α_z, κ)



The outer curve $\sqrt{2 \times \text{rel entropy}}$ is 0.0707 and for the inner curve it is 0.0354. The small diamond in the model depicts the approximating model.

Distorted Growth Rate Drifts

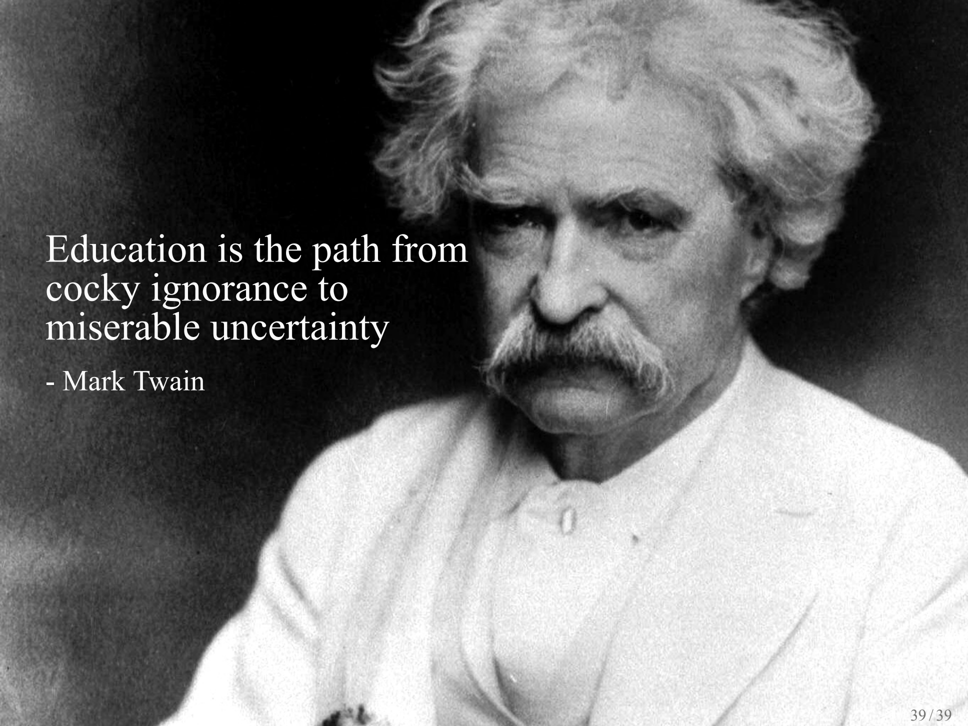


larger benchmark entropy | smaller benchmark entropy

black: approximating model; red: worst benchmark model; blue: lesser concern for misspecification; green: greater concern for misspecification

Conclusion

- Uncertainty prices fluctuate because investors struggle as they speculate about the future; not due to exogenously imposed stochastic volatility.
- Investors worry both about adverse shocks and their impact in future periods.


Education is the path from cocky ignorance to miserable uncertainty
- Mark Twain