Uncertainty Prices

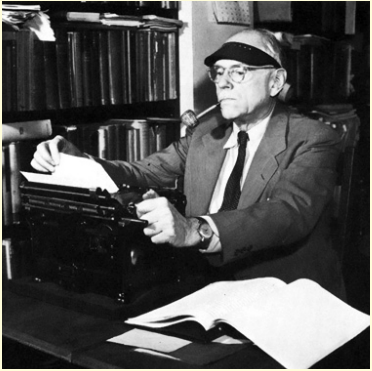
when Beliefs are Tenuous

Lars Peter Hansen (University of Chicago)

Thomas J. Sargent (New York University)

BI SoF Conference
June 3, 2016

Frank Knight and Uncertainty (1921)



"We must infer what the future situation would be without our interference, and what changes will be wrought by our actions. Fortunately, or unfortunately, *none* of these processes is *infallible*, or indeed ever *accurate* and *complete*."

Secular Stagnation?

Larry Summers, February 2015:
"The nature of macroeconomics has changed dramatically in the last seven years. Now ... concern is focused on avoiding secular stagnation."

Ben Bernanke, March 2015:
"Does the U.S. economy face secular stagnation? I am skeptical, and the sources of my skepticism go beyond the fact that the U.S. economy looks to be well on the way to full employment today."

Robert Gordon and Joel Mokyr recent debate on long-term growth prospects.

Private sector faces uncertainty about future macroeconomic growth.

Main ideas

Aim:

- Use dynamic variational preferences as a tool to study consequences of fears of model misspecification.

Approach:

- Surround a family of baseline models with a set that include statistically similar models
- Calibrate the size of the set using a measure of statistical discrimination
- Study how concerns about misspecification alter market prices of uncertainty

Application

Ingredients:

- Family of baseline models for log consumption growth with a predictable growth state variable
- Set of alternative less structured models
- Tractable robust decision problems for planner and representative investor

Outcome:

- Endogenous source of uncertainty price variation

Evidence from Financial Market Data

Private sector observation: Risk-On Risk-Off

- Investors' appetites for risk rise and fall over time

Academic research: Time-varying expected returns

- Measured risk-return tradeoffs from financial markets fluctuate over time
- "Risk-prices" are bigger in magnitude sometimes than others

This evidence poses a challenge to model builders: what explains these movements?

Cast of characters

- Baseline models and nonparametric alternatives
- Positive martingales that represent alternative probabilities
- Penalize deviations from the baseline models using statistical discrimination

References

- Anderson, Hansen, and Sargent, 2003, “A Quartet of Semigroups for Model Specification, Robustness, Prices of Risk, and Model Detection.” *JEEA*.
- Chen and Epstein, 2002, “Ambiguity, Risk, and Asset Returns in Continuous Time.” *Econometrica*.
- Gilboa and Schmeidler, 1989, “Maxmin Expected Utility with Non-unique Prior.” *Journal of Mathematical Economics*.
- Maccheroni, Marinacci and Rustchini, 2006, “Dynamic Variational Preferences.” *Journal of Economic Theory*.
- Petersen, James, and Dupuis, 2000, “Minimax Optimal Control of Stochastic Uncertain Systems with Relative Entropy Constraints,” *Automatic Control, IEEE Trans. Aut. Control*
- Newman and Stuck, 1979, “Chernoff Bounds for Discriminating between Two Markov Processes,” *Stochastics*.

Starting point

- Initial model parameterized by $\hat{\mu}, \hat{\phi}, \hat{\beta}, \hat{\kappa}, \alpha, \sigma$

$$dY_t = (.01)\left(\hat{\alpha}_y + \hat{\beta} Z_t\right) dt + (.01)\sigma_x \cdot dW_t$$
$$dZ_t = \hat{\alpha}_z dt - \hat{\kappa} Z_t dt + \sigma_z \cdot dW_t$$

- W a Brownian motion
- Think of Y as log consumption and use logarithmic utility
- Z generates "long-run risk"

Alternative models

- martingale:

$$M_t^H = \exp\left(\int_0^t H_u \cdot dW_u - \frac{1}{2}\int_0^t H_u \cdot H_u du\right).$$

- Implied perturbed probabilities

$$E^H\left[B_t|\mathcal{F}_0\right] = E\left[M_t^H B_t|\mathcal{F}_0\right]$$

- Implied perturbed evolution of W:

$$dW_t = H_t dt + dW_t^H$$

where dW_t^H is a standard Brownian increment under the H probability measure.

Normal shocks dW_t with history dependent distortions H_t to the drift

A family of parametric alternative models

- $$dY_t = .01\left(\alpha_y + \beta Z_t\right)dt + .01\sigma_y \cdot dW_t^R$$
$$dZ_t = \alpha_z dt - \kappa Z_t dt + \sigma_z \cdot dW_t^R$$
- Construct M^R where $R_t = \eta(Z_t) \equiv \eta_0 + \eta_1 Z_t$
where
$$\sigma = \begin{bmatrix} (\sigma_y)' \\ (\sigma_z)' \end{bmatrix},$$
and
$$\sigma\eta_0 = \begin{bmatrix} \alpha_y - \hat{\alpha}_y \\ \alpha_z - \hat{\alpha}_z \end{bmatrix}$$
$$\sigma\eta_1 = \begin{bmatrix} \beta - \hat{\beta} \\ \hat{\kappa} - \kappa \end{bmatrix}$$

Possible types of misspecification

- Benchmark models are parametric alternatives including time-varying specifications. Represent using martingales parameterized by R's and restricted via:

$$R_t \cdot R_t \leq \xi(Z_t).$$

- Represent other statistically similar probabilities represented by martingales parameterized by H's and restrained using a relative entropy penalization.

Discrepancy measure for martingales

- Relative entropy of M^H with respect to M^R

$$\begin{aligned}\Delta(M^H; M^R|z) =& \delta \int_0^\infty \exp(-\delta t) E\left[M_t^H \left(\log M_t^H - \log M_t^R \right) \Big| Z_0 = z \right] dt \\ =& \frac{1}{2} \int_0^\infty \exp(-\delta t) E\left(M_t^H |H_t - R_t|^2 \Big| Z_0 = z \right) dt.\end{aligned}$$

- Penalization

$$\theta \Delta(M^H; M^R|z)$$

where $\theta > 0$ is a penalization parameter.

Equilibrium construction

- Solve a “robust planners problem”
- Deduce restrained worst case model
- Compute shadow prices including the price of uncertainty

Outcome: worst-case probability $H_t^* = \eta^*(X_t)$ where $\eta^*(x) = \eta_0^* + \eta_1^* x$ within the parametric class and:

$$\underbrace{\text{“local risk price”}}_{} = \underbrace{.01\alpha}_{\text{risk price}} + \underbrace{(-H_t^*)}_{\text{uncertainty price}}$$

Robust planners problem

$x = (y, z)$ and m is a martingale realization. $\hat{\mu}(x)$ is the composite drift, U measures utility and δ captures the subjective discount rate.

- Approach one

$$0 = \min_{h,r} -\delta m V(x) + mU(x) + m\hat{\mu}(x)' \frac{\partial V}{\partial x}(x) + mh'\sigma' \frac{\partial V}{\partial x}(x) + \frac{m}{2} \text{trace}\left[\sigma' \frac{\partial^2 V}{\partial x \partial x'}(x)\sigma\right] + \frac{\theta m}{2}|h - r|^2.$$

  subject to $|r|^2 \leq \xi(x)$ where $mV(x)$ is the value function.

- Approach two

$$0 = \min_{h,r} -\delta m V(x) + mU(x) + m\hat{\mu}(x)' \frac{\partial V}{\partial x}(x) + mh'\sigma' \frac{\partial V}{\partial x}(x) + \frac{m}{2} \text{trace}\left[\sigma' \frac{\partial^2 V}{\partial x \partial x'}(x)\sigma\right] + \frac{\theta m}{2}|h - r|^2 + \frac{\kappa m}{2}\left[|r|^2 - \xi(x)\right].$$

  where κ is a Lagrange multiplier for an *ex ante* constraint and $mV(x)$ is the value function.

Calibrating robustness

A stylized model selection problem:

- Discriminate between two models:
  (i) a baseline model;
  (ii) an implied worst-case model.
- Given time series data, choose a model as either a Bayesian or max-min decision maker.
- Compute *ex ante* probabilities of making Type I and Type II errors as sample size increases.
- Use large deviation methods and equate decay rates in error probabilities.
- Convert decay rates into half lives.

Statistical calibrations

Hansen-Heaton-Li VAR style model matched to quarterly consumption data and data on business related income and personal dividends projected onto our parametric class

- project onto our parametric class by matching long-term implications
- correlation between the shock to consumption and the shock to the growth rate in consumption $\sigma_y \cdot \sigma_z \neq 0$

Aggregate income relative consumption

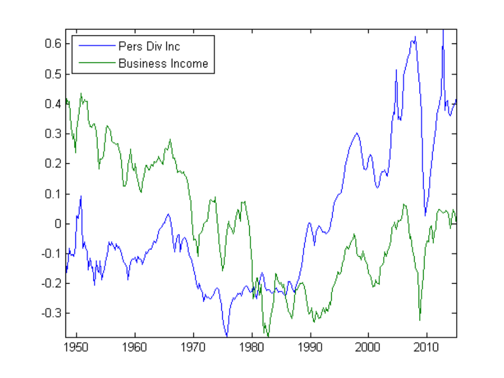
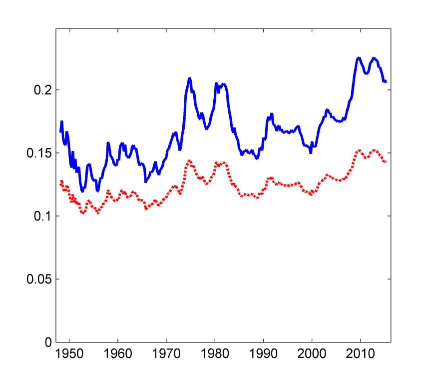


Figure: Business income is measured as corporate profits plus proprietor income. Personal dividend income is the aggregate dividends paid to individuals.

Worst-case parameter values

Half-Life	α_y	α_z	β	κ	$\underline{\kappa}$	$\alpha_y - \hat{\alpha}_y + \frac{\beta\alpha_z}{\kappa}$
∞	.386	0	1	.019	.019	0
80	.316	-.0072	1.024	.015	.0075	-.550
80	.322	-.0068	1.038	.013	.0050	-.593
80	.332	-.0061	1.058	.010	.0025	-.682

Uncertainty Price Time Series



Uncertainty prices to the two shocks predicted from estimation with aggregate data and inferred without direct use of asset market return data.

Conclusion

- Uncertainty prices fluctuate because investor struggles as they speculate about the future and not because of exogenously imposed stochastic volatility.
- Investors worry both about adverse shocks and how these shocks could be transmitted in the future.